Homework #3

1. From the formula for the gravitational force and the planet's acceleration, you can find out that the orbital motion's period relates to the semimajor axis and the central object's mass as in the following equation:

$$P^2 = \frac{4\pi^2 a^3}{GM} \quad (1)$$

$$P_\oplus^2 = \frac{4\pi^2 a_\oplus^3}{GM_\odot} \; (Apply\ to\ Earth) \quad (2)$$

Dividing the first formula by the second one, you will have:

$$P'^2 = \frac{a'^3}{M'} \quad (3)$$

P', a' and M' are in the units of the tropical year, AU and Solar mass, respectively. Your answer should be close to this: $M_{Jupiter} \approx 9.5{\times}10^{-4} M_{Sun} = 1.9{\times}10^{27} kg$.

2. $R_{Moon} = 384,000km$, $P_{Moon} = 27.3days = 2.36{\times}10^6 secs$. The acceleration is:

$$a = \frac{v^2}{R} = \frac{(2\pi R/P)^2}{R} = \frac{4\pi^2 R}{P^2} \quad (4)$$

Using corresponding values for Moon, you get $a_{Moon} = 2.72{\times}10^{-3} m{\cdot}sec^{-2}$. Compared to the acceleration on the surface of the Earth ($9.8\ m{\cdot}s^{-2}$), it's $2.78{\times}10^{-4}$ times smaller. It's because that the distance from the center of the Earth to the Moon ($R_{Moon} = 384,000km$) is about 60 times longer than that to the surface of the Earth ($R_{Earth} = 6,400km$), and according to the Newton's law, the acceleration is propotional to the square of inverse distance, which is $\frac{1}{60}{\times}\frac{1}{60} = 2.78{\times}10^{-4}$.

3.

$$v^2 = \frac{GM}{R} \quad (5)$$

$$\begin{aligned} v &= \sqrt{\frac{GM}{R}} && (6)\\ &= v_{\oplus} && (7)\\ &= 30\ km{\cdot}s^{-1}\ (same\ as\ Earth's\ orbital\ velocity) && (8)\\ E_k &= \frac{1}{2}mv^2 = 0.5{\times}1000{\times}30,000^2 = 4.5{\times}10^{11}\ Joules && (9)\\ E_p &= -\frac{GMm}{R} = -v^2{\times}m = -30,000^2{\times}1000 = -9{\times}10^{11}\ Joules && (10) \end{aligned}$$

The minus sign in front of the potential energy is because we usually define the potential energy is zero at the infinity. When the object moves against the force, the potential energy increases. The potential energy decreases when the object moves along the force, cause it loses the power to generate energy by doing so.

Then, your distance to Sun is increased by a factor of two, from the formula, you can see that your speed will decrease by a factor of $\sqrt{2}$ ($21.2\ km{\cdot}s^{-1}$). The kinetic energy will drop by a factor of 2 ($2.25{\times}10^{11}\ Joules$) and the potential energy becomes $-4.5{\times}10^{11}\ Joules$.

From the initial state to the current state, the total energy changes from $E_k + E_p = -4.5{\times}10^{11}\ Joules$ to $-2.25{\times}10^{11}\ Joules$, so the total energy you gain from the rocket engine is $2.25{\times}10^{11}\ Joules$. The total energy is conserved when you include the kinetic energy, the potential energy and the work done by the rocket.