Homework #11

1. a). Assuming the spherical geometry. With a constant density ρ, the mass of a sphere of radius r is the product of the density and the volume (V): $M_r = \rho \times V = \frac{4\pi}{3}\rho r^3$. So the mass is propotional to the density and the 3rd power of radius.

b). $v_r = \sqrt{\frac{GM_r}{r}} = \sqrt{\frac{4\pi}{3}\frac{G\rho r^3}{r}} = \sqrt{\frac{4\pi}{3}G\rho r^2} \propto r$. The orbital speed is propotional to the radius from the center of the sphere.

c). The orbital period $P = \frac{2\pi r}{v_r} = \frac{2\pi r}{\sqrt{\frac{4\pi}{3}G\rho r^2}} = \frac{\sqrt{4\pi^2 r^2}}{\sqrt{\frac{4\pi}{3}G\rho r^2}} = \sqrt{\frac{3\pi}{G\rho}}$

d). It's called solid-body rotation or rigid-body rotation because the whole thing rotates like a solid body. Every part of the body has the same period and doesn't have relative motion in respect to each other. Its opposite term is the "differential rotation".

e). In a solid-body rotation situation, we won't see any Doppler effect, because there is no relative motion among different parts. The distance between any two parts never changes, thus the relative velocity component along the connecting line of two parts is always zero.

2. a). Using the method in Homework #9, we get the distance of delta Cepheid is $D = \frac{1}{0.003}\ pc \sim \mathbf{333.3(pc)}$.

b). Using the period-luminosity relation, we know that the Cepheid variable (star A) with a period of **5** days is **10** times fainter than the variable (star B) with a period of **50** days, thus A is **2.5** greater than B in (apparent) magnitude. Since B has a mean magnitude of **24**, A would have a magnitude of **26.5**.

c). Delta Cepheid has a magnitude of **4**. Star A has a magnitude of **26.5**. Since $m \propto -2.5 log_{10} F$, $\Delta m = m_1 - m_2 = -2.5(log_{10}F_1 - log_{10}F_2) = 2.5 log_{10}(\frac{F_2}{F_1})$. Thus $\frac{F_2}{F_1} = 10^{\frac{\Delta m}{2.5}}$. Star A is 22.5 magnitudes fainter than delta Cepheid, so it's $10^{\frac{22.5}{2.5}} = \mathbf{10^9}$ times fainter in flux than delta Cepheid.

d). Because Star A has the same intrinsic luminosity as delta Cepheid does (according to the period-luminosity relation), the reason it appears fainter is that it is farther in distance. Based on the distance square relation ($F = \frac{L}{4\pi r^2}$), it is $\sqrt{10^9} = 10^{4.5} = \mathbf{31623}$ times farther in distance than delta Cepheid.

e). From answer a, we can get the distance to M100 is $333.3(pc) \times 31623 = 10540925.5(pc) \sim \mathbf{10(Mpc)}$.

f). $\frac{\delta\lambda}{\lambda} = \frac{v}{c} = \mathbf{0.004}$, so $v = 0.004c = 0.004 \times 3 \times 10^5 (km \cdot s^{-1}) = \mathbf{1,200(km \cdot s^{-1})}$.

g). From the Hubble's relation $v = H \times r$, we get $H = \frac{v}{r} \sim \mathbf{120}(\frac{\mathbf{km \cdots^{-1}}}{\mathbf{Mpc}})$.

3. a). The cosmic microwave backgroud (CMB) radiation of today's universe is the blackbody radiation of temperature **3K** (or 2.7K, if you like). The typical photon energy of a blackbody can be estimated by the relation $h\nu = \frac{hc}{\lambda} \sim kT$, where k is Stefan-Boltzmann constant. so $\lambda \sim \frac{1}{T}$. Because the temperature (T) has droped by a factor of $\frac{3000}{3} = \mathbf{1000}$ since the radiation was emitted, we can infer that the wavelength (space) has been stretched by the same factor. The volume has been stretched by a factor of $1000^3 = \mathbf{10^9}$.

b). The volume containing the same amount of matter was 10^9 times smaller than today's value, so the density was 10^9 times greater, or $10^9 \times 10^{-28}\ kg\ m^{-3} = 10^{-19}(kg \cdot m^{-3})$ at the time the radiation was emitted.

c). $10^{-19}(kg \cdot m^{-3}) = 10^{-19}(kg \cdot 10^{-6} cm^{-3}) = \mathbf{10^{-25}(kg \cdot cm^{-3})}$. It corresponds to $\frac{10^{-25} kg \cdot cm^{-3}}{1.67 \times 10^{-27} kg\ per\ proton} \sim \mathbf{60}$ *protons per cubic centmeter*.