

Homework #10

1. The orbital speed at the distance of r from an object of mass M can be found through the equation: $v = \sqrt{\frac{GM}{r}}$, where G is the gravitational constant.

a). Your orbital speed around the black hole is:

$$v = \sqrt{\frac{GM_{BH}}{r}} = \sqrt{\frac{5GM}{r}} = \sqrt{\frac{5 \times 6.67 \times 10^{-11} (Nm^2 kg^{-2}) \times 2 \times 10^{30} (kg)}{750,000 (m)}} \approx 2.98 \times 10^7 (m \cdot s^{-1})$$

b). The gravitational force can be calculated from the formula $F = \frac{GMm}{r^2}$. Assuming the weight is $m = 80 \text{ kg}$.

The force on feet is:

$$F_{feet} = \frac{GM_{BH} 0.5m}{r_{feet}^2} = \frac{6.67 \times 10^{-11} (Nm^2 kg^{-2}) \times 5 \times 2 \times 10^{30} (kg) \times 40 (kg)}{(749,999 (m))^2} = 4.743124 \times 10^{10} (N).$$

The force on head is:

$$F_{head} = \frac{GM_{BH} 0.5m}{r_{head}^2} = \frac{6.67 \times 10^{-11} (Nm^2 kg^{-2}) \times 5 \times 2 \times 10^{30} (kg) \times 40 (kg)}{(750,001 (m))^2} = 4.743111 \times 10^{10} (N).$$

c). The tidal force is:

$$F_{tide} = (4.743124 - 4.743111) \times 10^{10} (N) = 130,000 (N) \approx 286,600 (pounds).$$

Nobody wants to do that, the huge tidal force will pull the body apart.

2. a). With a constant acceleration a , starting from the zero speed, an object will reach a speed $v = at$ after time t , so it will take:

$$t = \frac{c}{1g} = \frac{3 \times 10^8 (m \cdot s^{-1})}{9.8 (m \cdot s^{-2})} = 30,612,245 (s) = 0.97 (yrs)$$

to accelerate up to the speed of light.

b). The distance from us to the center of the Milky Way is $8kpc$ (pp607). We can convert that into value in the unit of lightyear (ly), remember that $1ly$ is the distance you can travel in one year with the speed of light. $8kpc = 8,000 \times 3.3(ly) = 26,400(ly)$, which means it will take 26,400 years to travel this far at the speed of light. Now we are travelling at the speed $0.99c$, 1% slower, so we need a little more time, that's $\frac{100}{99} \times 26,400 = 26,666.7(yrs)$.

c). The slowing factor is:

$$\gamma = [1 - \frac{v^2}{c^2}]^{-1/2} = \sqrt{[1 - \frac{v^2}{c^2}]^{-1}} = \sqrt{[1 - \frac{(0.99c)^2}{c^2}]^{-1}} = \sqrt{[1 - 0.99^2]^{-1}} = 7.09.$$

So the trip will still last about $26,666.7/7.09 = 3,761.8(yrs)$, we definitely don't want to try it.