

Homework #1

1. Because Mars' orbital period (1.88 yrs) is longer than Earth's orbital period (1.00 yr), Earth moves faster than Mars does around Sun, so next time when Mars is at its closest approach to Earth, if Mars makes n times along the orbit, Earth must make $n+1$ times, the total time (T) needed to do it is $n \times 1.88 \text{ yr}$ for Mars and $(n + 1) \times 1.00 \text{ yrs}$ for Earth, and they are equal, so we have:

$$1.88n = 1.00(n + 1) \quad (1)$$

$$0.88n = 1.00 \quad (2)$$

$$n \approx 1.14 \quad (3)$$

so, the total time $T = n \times 1.88 = (n+1) \times 1.0 \approx 2.14 \text{ yrs}$. Or, in a century, Earth makes 100 times around Sun, Mars only makes $\frac{100}{1.88} \approx 53.19$ times, so Earth catches up (meets) Mars $100 - 53.19 = 46.81$ times during one century. The separation between two adjacent meets is $100 \div 46.81 \approx 2.14 \text{ yrs}$.

3. In the winter, days are shorter than nights on North hemisphere, especially at the full moon around December 22, when Sun is at its southernmost point on the Ecliptic, Winter Solstice, the night is longest on North hemisphere, about 4 hours longer than daytime for Austin (at altitude about 30°). Because it is full moon, so Moon is on the opposite side of Sun on the ecliptic plane. When Sun sets south of the west of the sky around 5pm, Moon rises north of the east. Moon moves across the sky in a plane which has an angle of $60^\circ = 90^\circ - 30^\circ$ from the south of the horizon. Around 7am, Moon sets north of the west. In June, Sun is at its northernmost point on the Ecliptic, Summer Solstice, the night is shortest. Moon rises from south of the east around 7pm and sets at south of the west around 5am.

2. Venus's rotational period (243 days) is longer than its orbital period (225 days), at the point 1 (referring the attached figure), A is looking at Sun at noon, next time A looking at Sun at noon happens before Venus returns back to point 1, say point 2. Angle α is caused by its revolution of the center of Venus, angle β is caused purely by Venus' rotation about its axis, the time which is taken for all these happening is T . We have:

$$T/243 = \beta/360, \text{ because Venus rotates } 360^\circ \text{ in } 243 \text{ days} \quad (4)$$

$$T/225 = \alpha/360, \text{ because Venus revolves } 360^\circ \text{ in } 225 \text{ days} \quad (5)$$

$$\frac{\alpha}{360} + \frac{\beta}{360} = \frac{T}{243} + \frac{T}{225} \quad (6)$$

$$\frac{\alpha + \beta}{360} = T \times \frac{243 + 225}{243 \times 225} \quad (7)$$

$$\alpha + \beta = 360, \text{ from geometry} \quad (8)$$

$$T = \frac{243 \times 225}{243 + 225} = 116.8 \text{ days} \quad (9)$$

Another way to do it is: Sun moves $360^\circ/225 = 1.6^\circ$ per Earth day on the sky because of Venus' orbital motion only. It moves $360^\circ/243 \approx 1.48^\circ$ per Earth day on the sky because of Venus' rotation only. These two motions (of Sun) are in the same direction seen from the surface of Venus, so each Earth day, Sun moves $1.48^\circ + 1.6^\circ = 3.08^\circ$ on the sky of Venus, it takes $360/3.08 \approx 116.8$ Earth day for Sun to complete 360° around Venus.

