ASTRONOMY 301
PROBLEM SET NUMBER 4
DUE IN CLASS
Monday, November 24, 2003

Read these instructions first. Print your name on each page of your answer sheets and staple them together. It is important to show the reasoning you used to solve the problem and any formulae you used and algebraic manipulation that you did. You are encouraged to work together to figure out the method of solution, but you must write out the solutions independently in your own words. Do all five problems; each problem counts 1 point. Turn in your answers in class on the due date.

1. An open cluster of stars has a main sequence from B5 down to M5, but no stars hotter than B5. If the mass of a B5 star is 6 solar masses, what is the age of the cluster? Use the relation between lifetime and mass given in class.

2. The speed of the Sun in its orbit about the Galactic center is 220 km/sec. Its distance from the center is 8500 pc. Assuming that the Sun's orbit is circular, calculate the mass of the Galaxy (in solar masses) interior to the Sun's orbit from Kepler's Third Law. Recall that the circumference of a circle is $2 \pi r$, where r is the radius. Remember that 1 pc is 206265 AU and 1 AU is 1.5×10^8 km.

3. A Cepheid variable star is observed in the nearby galaxy M31. Its period of variation is measured to be 40 days. Using the period - luminosity relation given in the table below, what is the luminosity of the Cepheid in solar luminosities?

Period (days)	Luminosity (solar luminosities)
2.5	645
4.0	1120
6.3	2000
10.0	3550
15.8	6310
25.1	11500
40.0	20900
63.1	38000

4. For the Cepheid in M31, what is its luminosity in ergs/sec? (Recall that the solar luminosity is 3.85×10^{33} ergs/sec.)

5. The Cepheid in M31 has its apparent brightness measured at 1.06×10^{-12} ergs/sec/cm^2. The inverse square law of light may be written as

$$b = L \div 4 \pi r^2$$

Where b is the apparent brightness, L is the luminosity and r is the distance. How far away is the Cepheid in centimeters? Convert your answer to parsecs by knowing that one parsec contains 3.09×10^{18} centimeters.

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SOLUTIONS

#1. We know that a star has a lifetime that is determined by its mass. The relation given in class for this is

$$\text{lifetime} = \text{mass}^{-2.5} \times 10^{10} \text{ years}$$

For a B5 star with a mass of 6 solar masses, we compute $6^{-2.5} \times 10^{10}$ years. On some calculators, this is straightforward to do, on others it is necessary to take the roundabout calculation of

$$\text{Lifetime} = \text{mass}^{-2}\ \text{mass}^{-0.5} \times 10^{10} \text{ years}$$

which is the same as

$$\text{Lifetime} = 1/M^2 \times 1/\text{sqrt}(M) \times 10^{10} \text{ years}$$

Substituting the value 6 solar masses, we get 0.0113×10^{10} years, i.e., 113 million years. Because the cluster stars all formed at the same time and because the stars more massive that B5 have died, the cluster must be 113 million years old.

#2. We know that the Sun's speed is 220 km/sec in an orbit that has a semimajor axis of 8500 pc. To get the period we must compute the circumference of the orbit and divide that by the speed. First we convert the semimajor axis from parsecs to km by multiplying by 206265 (to go from pc to AU) then by 1.5×10^8 (to go from AU to km) getting 2.63×10^{17} km.

The circumference of a circle is $2 \pi r^2$, so the circumference is

$$\text{circumference} = 2 \pi (2.63 \times 10^{17}) \text{ km} = 1.65 \times 10^{18} \text{ km}$$

Dividing by the speed, 220 km/sec, we get 7.5×10^{15} sec. To convert this to years, we divide by $365.25 \times 24 \times 60 \times 60 = 3.15 \times 10^7$ sec/yr, obtaining 238×10^6 years.

To compute the mass of the Galaxy interior to the Sun's orbit, we use Kepler's Third Law in solar units

$$(M_1 + M_2)P^2 = a^3 ,$$

where M_1 is the Galaxy's mass in solar masses, M_2 is the Sun's mass which is entirely negligible, P is the Sun's period of revolution in years, and a is the Sun's orbital semimajor axis in AU. Putting the Sun's mass to zero,

$$M_1 P^2 = a^3$$

Put in $P = 238 \times 10^6$ years, $a = 8500$ pc $\times$ 206265 AU/pc and we get

$$M_1 = (1.75 \times 10^9)^3 \div (238 \times 10^6)^2 \text{ solar masses}$$

$M_1 = 9.46 \times 10^{10}$ solar masses for the mass of our Galaxy.

#3. A Cepheid in M31 has a period of 40 days. To get its luminosity in solar, luminosities we look it up in the period-luminosity relation: 20900 = 2.09×10^4 solar luminosities.

#4. The above Cepheid has a luminosity of 2.09×10^4 solar luminosities. One solar luminosity is 3.85×10^{33} ergs/sec,. We multiply these together to get the Cepheid's luminosity in ergs/sec: 8.05×10^{37} ergs/sec.

#5. The above Cepheid has an apparent brightness of 1.06×10^{-12} ergs/sec/cm$^{2.}$ and a luminosity of 8.05×10^{37} ergs/sec. The inverse square law for light is

$$b = L \div 4 \pi r^2$$ where *r* is the distance in cm.

This can be rearranged to solve for *r*,

$$r^2 = L \div (4 \pi b) \quad cm^2$$

Inserting the known values, we get

$$r^2 = 6.04 \times 10^{48} \text{ cm}^2$$

Taking the square root, $r = 2.46 \times 10^{24}$ cm

Converting this to parsecs by dividing by the number of cm in a pc, 3.09×10^{18} cm,

$$R = 7.96 \times 10^5 \text{ parsecs}$$

The galaxy M31 is 796 kpc from the Sun.