The Frequency Splitting of Multiplets in the Oscillations of Weakly Magnetic White Dwarf Stars

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ABSTRACT

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This research is an attempt to ascertain how the frequencies of nonradial g-mode oscillation of the sort observed in white dwarf stars are affected by a magnetic perturbation. Specifically, we have focussed on the differences in frequency between oscillations with spherical harmonic indices of the same ℓ but different m. In general, we have found that this 'splitting' of multiplets is affected by a magnetic field with a global structure. This has been demonstrated through the derivation of approximate analytical expressions and by numerical calculations based on exact formulae for particular forms of the magnetic field. From these calculations, we were able to determine the approximate field

strength, ~ 1050 G, which would lead to the average observed splitting asymmetries in $\ell = 1$ modes in the DBV GD 358, as given in Winget et al. (1994). We then attempted to extract the radial variation of the magnetic field from the variations in the splittings as a function of radial overtone number, using either a constant field or a locally disordered field, but could not obtain a self-consistent solution in either case. This implies that either our model of the equilibrium structure of the star is inadequate, or that the formalism presented here is too simplistic.

Table of Contents

List of Figures

Chapter 1

Introduction

Of the few astronomical bodies we can examine closely, many exhibit the phenomenon of a magnetic field. For instance, the Earth, Jupiter, and the Sun all exhibit measureable fields. The underlying mechanisms by which these fields are formed and maintained, however, is largely a mystery. It is quite likely that there is a relationship between the rotation of such objects and their magnetic fields. Although dynamo theory offers a phenomenological description of the generation and evolution of magnetic fields, it is not a theory which is presently derivable from the underlying microphysics. Given this state of affairs, a new tool is needed which allows us to obtain more information about the properties of magnetic fields and rotation, not just at the surface, but within stellar objects.

Asteroseismology is just such a tool. Asteroseismology uses a knowledge of the normal modes in model systems to fit observations of the frequency spectrum of pulsating stars. Since a normal mode is a property of the system as a whole, this technique provides information not only about properties of the star at its surface, but also about its deeper layers. As an example, it is possible to find the thickness of surface hydrogen or helium layers on white dwarfs by examining the variations in the period spacings between consecutive radial overtones. It is our hope that it will similarly be possible to extract information about the magnetic fields and rotational curves of such objects.

Our optimism is not without reason. It happens to be the case that, in many important ways, white dwarfs are very simple objects. There are no complications from nuclear energy generation, and the convection zones are small in comparison with those in most other stars. They also have extremely high surface gravities, which confines motions mainly to nonradial directions. As a result, their equilibrium configurations are highly spherical. Thus, many important processes such as pulsation or convection, which are strongly present in other types of stars, are present at perturbational levels in white dwarfs. This fact, and the physical and mathematical power which comes with it, makes it likely that these important problems will first be understood in the context of white dwarf stars.

For example, our plan is to treat the magnetic field as a non-spherical perturbation to the spherical symmetry of the white dwarf. As such, it can lift the degeneracies between modes of the same frequency, much in the same way that the Zeeman effect lifts the degeneracy between atomic states of the same energy. The following chapter centers on a discussion of the properties of these normal modes, since it is through them that the eigenfrequencies are affected.

Chapter 2

General Properties of White Dwarf Oscillations

The particular kind of oscillations which are found to be excited in white dwarf stars are called 'g-mode' oscillations. These are modes in which the restoring force for each fluid element is gravity. A familiar example is a wave on the surface of water; if there were no gravity, the crest of the wave would not be pulled back toward the main body of water. Hence, gravity (in the form of differences in 'buoyancy') supplies the restoring force. On the other hand, sound waves need only a compressible medium through which to propagate; the restoring force for a fluid element comes from pressure variations. Modes for which pressure is the dominant restoring force are therefore called 'p-modes.' Since it is the g-modes which are excited to observable amplitudes in white dwarfs, we will confine our discussion to them in all that follows.

Ignoring such complicating effects as rotation or a magnetic field, white dwarfs have spherical symmetry. This makes it possible, as in many problems involving spherical symmetry, to represent the angular dependence of the eigenfunctions in terms of an expansion in spherical harmonics, $Y_\ell^m(\theta,\phi)$. In particular, the displacement vector for the oscillations is given by

$$\vec{\xi} = \left(\xi_r(r), \xi_h(r)\frac{\partial}{\partial\theta}, \xi_h(r)\frac{1}{sin(\theta)}\frac{\partial}{\partial\phi}\right) Y_\ell^m(\theta,\phi)e^{i\sigma t}, \qquad (2.1)$$

where the first, second, and third components of $\vec{\xi}$ are the $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ components, respectively, and σ is the angular frequency of the oscillation (Unno et al. 1989, equation 13.60). This has the effect of reducing the problem to a one-dimensional determination of the functions $\xi_r(r)$ and $\xi_h(r)$, which are the amplitudes of the radial and horizontal displacements. The number of nodes in ξ_r, denoted by k, when taken together with ℓ and m form a complete set of numbers with which to label the modes, i.e., they form a set of 'good quantum numbers.'

Since the surface gravities of white dwarfs are so large, it is in general easier to compress material through horizontal motions than it is to displace it vertically against gravity. This is especially true near the surface, where densities are low, so it is not surprising that $\xi_h \gg \xi_r$ near surface. This argument does not apply to p-modes, since the restoring force is pressure, not gravity.

The upper two plots in figure 2.1 show ξ_r and ξ_h for an $\ell = 1, k = 13$, mode, with a period of 616 seconds; the model for which the computations were done is a best fit model of the DB white dwarf GD 358, as described in Bradley and Winget (1994). The model has a mass of .61 $M_\odot$, a $T_{\rm eff}$ of 24,043 K, and a surface helium layer mass of $\sim 10^{-6} M_\star$. The pulsation code used is a Runge-Kutte-Fehlberg (RKF) integrator (Kawaler et al. 1985). As can be seen, $\xi_h/\xi_r \sim 1100$ near the surface. One consequence of this is that the luminosity variations are not due to the changing size of the disc, which is a major effect in Cepheid variables (Robinson et al. 1982). Rather, the compression of a region of the star's surface causes its temperature and therefore its luminosity to vary.

An additional point to notice about figure 2.1 is the relative magnitude of ξ and its radial derivatives. The lower two plots are of $\frac{d}{dr}(r\xi)$:

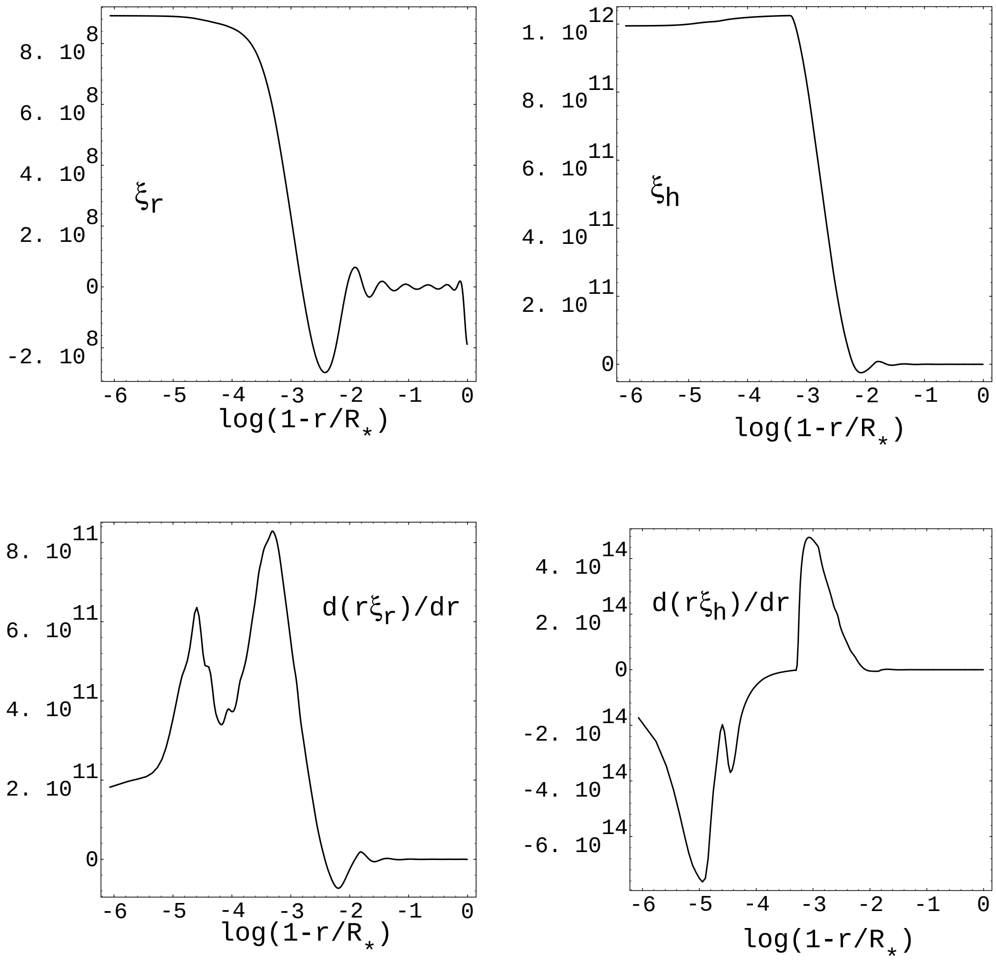

Figure 2.1: The components of ξ and their derivatives versus $\log(1 - r/R_\star)$, where $R_\star$ is the radius of the star. The x-axis is such that the surface is on the left and the center is on the right. It should be noted in these figures that $\xi_h \gg \xi_r$ near the surface, where by surface we mean the region in which $\log(1 - r/R_\star) < -2$.

$$\frac{d}{dr}(r\xi) \approx r\frac{d}{dr}\xi \approx \frac{r}{d}\xi \tag{2.2}$$

where d is a local scale height for the variation of ξ. We see that the ratio r/d has a magnitude of $\sim 10^2$–10^3. Thus, the largest of the four plots is

$$\frac{d}{dr}(r\xi_h) \approx r\frac{d}{dr}\xi_h. \tag{2.3}$$

We will use this fact later to derive approximate expressions for the perturbations to the eigenfrequencies.

Another important property of g-mode oscillations in white dwarf stars is that they are localized near the surface, i.e., the eigenfunctions are small in the core of the star. This can be realized from figure 2.1 by keeping in mind the logarthmic nature of the x-axis, e.g., $\log(1 - r/R_\star) = -2$ means $r = .99R_\star$. Essentially, this is due to the behavior of the Brunt-Väisälä frequency, denoted by N. The Brunt-Väisälä frequency is the natural frequency of oscillation of a displaced, buoyant fluid element (Unno et al. 1989). It is thus a local frequency for g-mode oscillations; when the the frequency of a g-mode is greater than the local Brunt-Väisälä frequency the mode is evanescent ('damped') in that region. Since the Brunt-Väisälä frequency becomes very small deep in the interior of a white dwarf, g-modes are evanescent there and have small amplitudes.

There is one final point to note about the eigenfunctions in connection with the magnetic field. In section 4.1, it is shown that the frequency shifts due to magnetic splitting are most sensitive to the radial derivative of the horizontal component of the eigenfunctions, that is, to $\frac{d}{dr}\xi_h$. In a convection zone, however, $\frac{d}{dr}\xi_h$ drops suddenly due to the sudden drop in the Brunt-Väisälä frequency, which becomes negative in the convection zone. This is shown dramatically in figure 2.2 for a $.6M_\odot$ pure carbon model. From asymptotic theory we obtain the following relation for the amplitude of ξ_h (from equation 15.4, Unno et al. 1989):

$$\xi_h \sim \exp\left(-\int_0^r \frac{N^2}{g}\right). \tag{2.4}$$

Thus, as N^2 drops near zero in the convection zone, ξ_h becomes nearly independent of r, so $\frac{d}{dr}\xi_h \sim 0$.

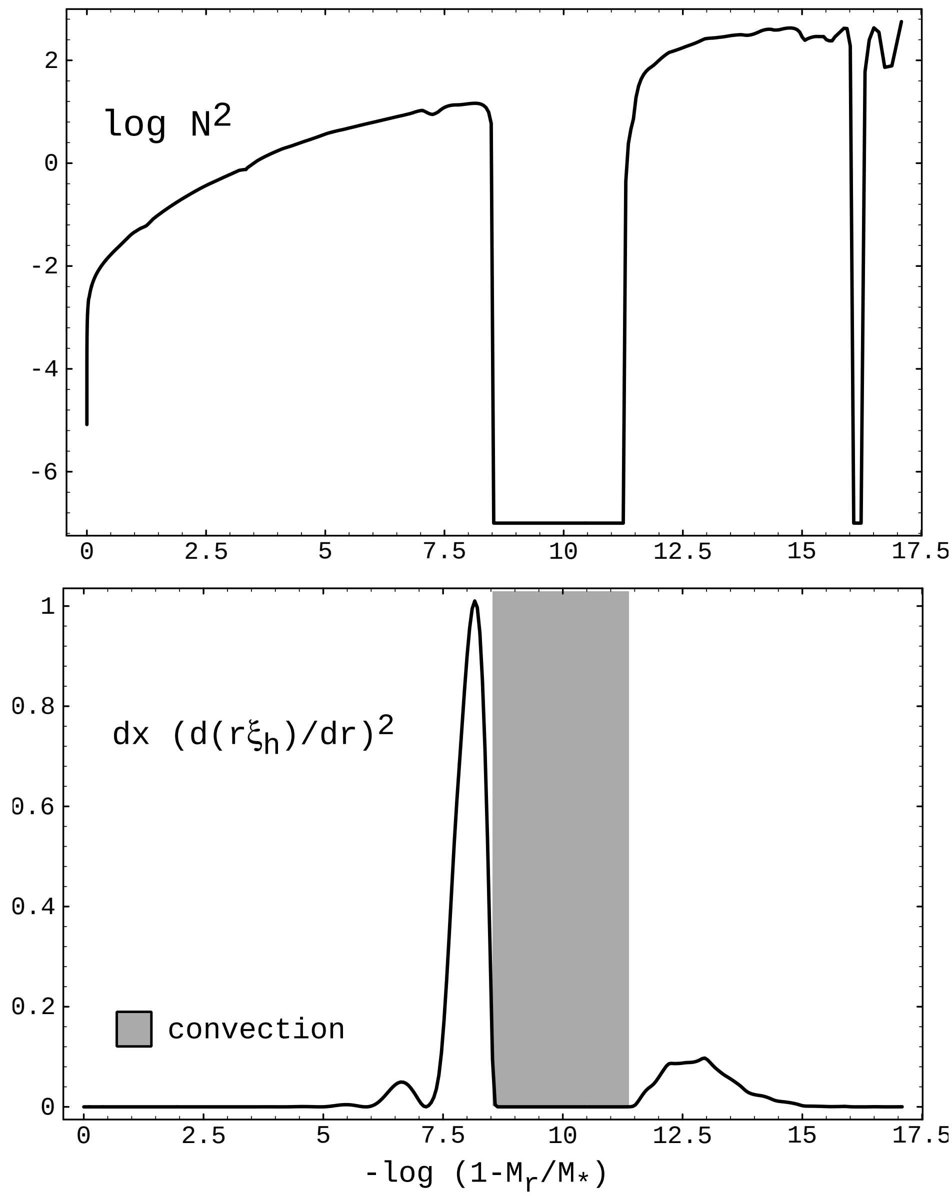

Figure 2.2: The lower plot is $dx\left(\frac{d}{dr}\xi_h\right)^2$ versus $x = \log(1 - M_r/M_\star)$, where the dx indicates the integral is now weighted linearly with the x-axis. The upper plot is the Brunt-Väisälä frequency. Clearly, as N^2 drops off in the convection zone, $\frac{d}{dr}\xi_h$ decreases rapidly as well.

Chapter 3

The Perturbation Formalism

To avoid confusion, we should note that there are two senses in which the word 'perturbation' is being used. The first use arises in describing the oscillations of the star about its equilibrium state, with the assumption that the amplitude of these oscillations is small. The second use is in our treatment of the magnetic field as a perturbation, both to the structure of the star and to its eigenfunctions and eigenfrequencies. In most instances in our discussion, 'perturbation' will refer to the latter of these two, since we are treating the eigenmodes of the star as essentially a solved problem.

The magnetic field affects the structure of the star in the same order that it affects the eigenfrequencies and eigenfunctions, i.e., it is not in principle suitable to use the equilibrium quantities in the non-magnetic case in order to calculate the the effect on the eigenfrequencies. This can be seen by an examination of the equations which govern the structure and pulsational properties of a star, equations 3.1 and 3.2, respectively:

$$0 = -\nabla\Phi - \frac{1}{\rho}\nabla p + \frac{1}{4\pi\rho}(\nabla \times \vec{B}) \times \vec{B} \tag{3.1}$$

$$\sigma^2\vec{\xi} = \mathcal{L}(\vec{\xi}) + \mathbf{B}(\vec{\xi}) \tag{3.2}$$

where the operators $\mathcal{L}(\vec{\xi})$ and $\mathbf{B}(\vec{\xi})$ are defined in Unno et al. (1989). It should be noted that $\mathbf{B}(\vec{\xi})$ is proportional to B^2, so that both equations are perturbed by order B^2. This situation may be contrasted with the treatment of rotation as a perturbation; the perturbation to the eigenfrequencies is linear in Ω due to the Coriolis term but the perturbation to the equilibrium structure is second order in Ω due to the centrifugal forces, and so may be neglected in a first order calculation. This is not necessarily true when treating a magnetic perturbation.

A second complication arises if the magnetic field does not vanish at the surface of the star. It will dominate the dynamics in the region very near the surface by virtue of the fact that the magnetic pressure $B^2/8\pi$ will greatly exceed the gas pressure. In this region, the eigenfunctions will be significantly affected, and a perturbational approach may not be valid. In the following sections it will be shown that the preceding difficulties are not in any way insurmountable for the cases we wish to consider.

3.1 Derivation of Frequency Splitting

We wish to calculate the difference in frequencies between a magnetic and a nonmagnetic system. The following approach is taken from Goosens et al. (1976). First, we rewrite equation 3.2 in these two cases:

$$\sigma'^2 \vec{\xi'} = \mathcal{L}'(\vec{\xi'}) + \mathbf{B}(\vec{\xi'}) \tag{3.3}$$

$$\sigma^2 \vec{\xi} = \mathcal{L}(\vec{\xi}), \tag{3.4}$$

where the primed quantities refer to the magnetic system and the unprimed quantities refer to the otherwise identical nonmagnetic system. We should also remember that $\mathcal{L}$ is a Hermitian operator. In addition, we are considering eigenfunctions $\vec{\xi}$ and $\vec{\xi'}$ which become identical in the limit that the magnetic field goes to zero.

Assuming that the frequencies are real, we take the inner product of $\vec{\xi}^*$ with equation 3.3, subtract from it the inner product of $\vec{\xi}'$ and the complex conjugate of equation 3.4, and integrate over the mass of the nonmagnetic configuration:

$$(\sigma'^2 - \sigma^2) \int \rho dV \vec{\xi}^* \cdot \vec{\xi}' = \int \rho dV \vec{\xi}^* \cdot \mathbf{B}(\vec{\xi}') + \int \rho dV (\vec{\xi}^* \cdot \mathcal{L}'(\vec{\xi}') - \vec{\xi}' \cdot \mathcal{L}(\vec{\xi}^*)). \quad (3.5)$$

Since $\mathcal{L}'$ is a function of equilibrium quantities in the magnetic case, we may write

$$\mathcal{L}' = \mathcal{L} + \delta\mathcal{L}, \quad (3.6)$$

where $\delta\mathcal{L}$ is due to the magnetic perturbation. Inserting this relation into equation 3.5 and using the hermiticity of $\mathcal{L}$, we obtain

$$(\sigma'^2 - \sigma^2) \int \rho dV \vec{\xi}^* \cdot \vec{\xi}' = \int \rho dV \vec{\xi}^* \cdot \mathbf{B}(\vec{\xi}') + \int \rho dV (\vec{\xi}^* \cdot \delta\mathcal{L}(\vec{\xi}')). \quad (3.7)$$

This equation embodies the two contributions to the frequency perturbations previously mentioned: the first term on the right-hand side (RHS) results from the effect of the magnetic field directly on the oscillations themselves, and the second is due to the changes in the equilibrium structure of the star. Both of these terms are quadratic in the magnetic field strength.

3.2 The Magnitude of the Structural Perturbations

We wish to examine the second term on the RHS of equation 3.2, which is due to perturbations of the equilibrium quantities. The quantity $\delta\mathcal{L}$ is significant only near the surface of the star, since this is the only region in which the magnetic pressure can dominate; we therefore expect this integral to be dominated by the surface region of the star. What we would like to show is that, for small magnetic fields, this term is negligible. Intuitively, this makes sense, at least for *extremely* small fields, since such a field should have essentially no effect, even though for it, too, there is a region in which the magnetic pressure dominates over the gas pressure. As it turns out, this is easier said than proven.

The problem arises because the region in which the magnetic pressure dominates is essentially infinite (at least until one worries about the breakdown of the fluid approximation at very low densities). Thus, it would appear to be necessary to look at the magnitude of the displacement in this region, in hopes that it would have a small enough amplitude to be safely ignored. There are two reasons that this might be appropriate. First, it may simply be that g-modes become non-propagating in this region. This seems plausible, since the local oscillation frequency of a displaced element in this region goes to infinity as the density goes to zero; therefore, there is a large difference between the frequency with which the wave is attempting to drive oscillations and the natural response of the medium. In addition, it is possible to think of the different velocities with which disturbances propagate in the two regions. As the g-mode travels upward in the atmosphere, it moves from a region of finite velocity to one with an arbitrarily large velocity (the Alfvén velocity goes to infinity as the density goes to zero), so we would expect significant reflection to occur at the 'boundary.' We have not made a detailed analysis of the dynamics in this region, so the answers to these questions remain open for now; they will be an integral part of our future investigations.

There is another way out of this dilemma; all we need to do is impose additional conditions on the equilibrium magnetic field, rather than considering completely general fields. In particular, if we merely assume that in the outer layers of the star that $\nabla \times \vec{B} = 0$ (which is equivalent to saying that there is no equilibrium current in these regions), then the perturbations to the pressure and density, and hence $\mathcal{L}$, are identically zero; the term simply goes away. This will be a valid assumption when the magnetic field is generated deeper within the star.

As an aside, there is an interesting effect which the magnetic field may have on the oscillations near the surface of the star, which is discussed in Biront et al. (1982). The main result of their calculation was that in a star which in the absence of a magnetic field would have purely radial oscillations, near the surface the oscillations would be bent by the magnetic field from the radial direction and would acquire a significant horizontal component. In a white dwarf, we are already considering nonradial pulsations, but the geometry of these pulsations would be greatly altered near the surface by this effect. This obviously would tend to mix the modes, and could lead to different brightness patterns on the star's surface than would otherwise be expected from the spherical harmonic angular dependence of the eigenfunctions. This last effect would be noticeable if the eigenfunctions became significantly altered at an optical depth of one or greater.

In figure 3.1, the optical depth to which a given strength magnetic field has a significant effect is plotted, where we have used the criterion

$$P_{\text{gas}} = P_{\text{mag}}, \tag{3.8}$$

where $P_{\rm mag} \equiv B^2/8\pi$. We see that a field of about 6000 G will alter the dynamics to an optical depth of one. Since we expect the displacements $\vec{\xi}$ in this region to be nearly parallel to $\vec{B}$, it would be possible in principle to see the global structure of the magnetic field. At the very least, modes of different ℓ would have the same angular dependence, so the geometric factor due to integrating over the disc of the star would be the same for modes of any ℓ. It is also possible that a magnetic field could act as a mirror and 'reflect' the eigenmodes before they had a chance to get close enough to the surface to be observed. This would also be a function of how deep inside the star this occurs, which would depend on the magnetic field strength. If this mechanism can be shown to occur, it would set limits on how large a magnetic field we are ever likely to observe in a pulsating white dwarf.

3.3 The Explicit Form of $\mathbf{B}(\vec{\xi})$

In order to obtain the first order corrections to the eigenfrequencies, we insert $\sigma' = \sigma + \Delta\sigma$ into equation 3.7 and keep terms of first order, with the result that

$$\Delta\sigma = \frac{1}{2\sigma} \cdot \frac{\int \rho dV \vec{\xi}^* \cdot \mathbf{B}(\vec{\xi'})}{\int \rho dV \vec{\xi}^* \cdot \vec{\xi'}}, \tag{3.9}$$

where we have ignored the structural perturbations, as was discussed in the previous section. Again, we note that $\vec{\xi'}$ is the eigenfunction in the magnetic system, which is nearly equal to $\vec{\xi}$ except near the surface.

The explicit form for $\vec{\xi}^* \cdot \mathbf{B}(\vec{\xi'})$ is

$$\begin{aligned} \vec{\xi}^* \cdot \mathbf{B}(\vec{\xi'}) &= \frac{1}{4\pi\rho} \, \vec{\xi}^* \cdot \Big[[\rho^{-1}(\vec{\xi} \cdot \nabla)\rho + \nabla \cdot \vec{\xi}] \vec{B} \times (\nabla \times \vec{B}) \\ &- [(\nabla \times \vec{B'}) \times \vec{B} + (\nabla \times \vec{B}) \times \vec{B'}] \Big], \end{aligned} \tag{3.10}$$

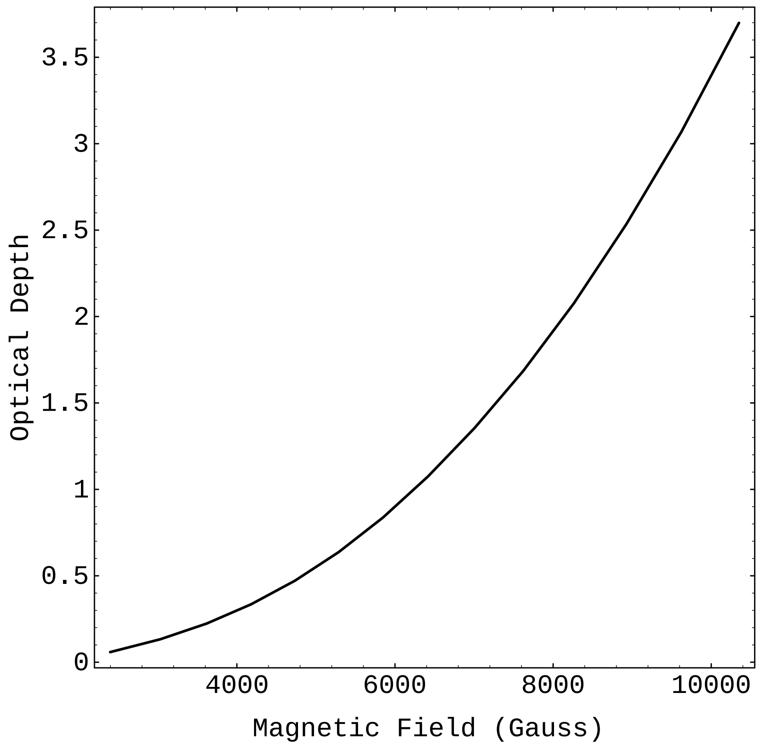

Figure 3.1: The optical depth to which a given magnetic field can significantly affect the eigenfunctions.

where $\vec{B'} = \nabla \times (\vec{\xi'} \times \vec{B})$ is the perturbation to the magnetic field due to the oscillations (from equation 19.13, Unno et al. 1989). Since we believe that $\vec{\xi} \parallel \vec{B}$ in the outer regions of the star, $|\vec{\xi'} \times \vec{B}| \to 0$ so $|\vec{B'}(\vec{\xi'})| \to 0$. Again, this is not the same statement as $|\vec{B'}(\vec{\xi})| \to 0$, because $\vec{\xi}$ and $\vec{\xi'}$ are different in this region.

The second term in the integral over $\vec{\xi^*} \cdot \mathbf{B}(\vec{\xi'})$ may be more conveniently expressed:

$$\int \rho dV \left(-\frac{1}{4\pi\rho}\right) \vec{\xi^*} \cdot ((\nabla \times \vec{B'}) \times \vec{B}) = \frac{1}{4\pi} \int dV (\nabla \times \vec{B'}) \cdot (\vec{\xi^*} \times \vec{B}) \quad (3.11)$$

$$= \frac{1}{4\pi} \int dV \vec{B}'(\vec{\xi'}) \cdot \vec{B}'(\vec{\xi}) - \frac{1}{4\pi} \int_S ds\, \hat{n} \cdot [(\vec{\xi}^* \times \vec{B}) \times \vec{B}'].$$

From previous arguments, the surface term vanishes. In the volume integral, we may safely replace $\vec{\xi'}$ with $\vec{\xi}$, so we find that this term simplifies to

$$\frac{1}{4\pi} \int dV |\vec{B}'|^2, \tag{3.12}$$

where the radial integration runs from the center to R_c, the point at which the magnetic pressure $B^2/(8\pi)$ equals the gas pressure. The integral is cut off at this point since we are assuming that $\vec{B}'$ approaches zero outside of this region. We next wish to explore the effects of particular magnetic field geometries.

Chapter 4

Magnetic Field Geometries

For convenience sake we have been suppressing the dependencies of the eigenfrequencies and eigenfunctions on k, the radial overtone number, and ℓ and m, the spherical harmonic indices. We will now write $\vec{\xi}_{klm}$ when we mean the mode in the nonmagnetic, spherically symmetric system which is composed of a single spherical harmonic. $\vec{\xi'}_{klm}$ will represent the mode in the magnetic system which approaches $\vec{\xi}_{klm}$ as the magnetic field is reduced to zero; this mode will not in general be composed of a single spherical harmonic. With this closer attention to detail, equation 3.9 becomes

$$\Delta\sigma_{klm} = \frac{1}{2\sigma_{kl}} \cdot \frac{\int \rho dV \vec{\xi}^{*}_{klm} \cdot \mathbf{B}(\vec{\xi'}_{klm})}{\int \rho dV \vec{\xi}^{*}_{klm} \cdot \vec{\xi'}_{klm}}. \tag{4.1}$$

The fact that the unperturbed frequencies, σ_{kl}, do not depend on m is simply due to the assumed spherical symmetry in the nonmagnetic problem. The main point of these calculations is to find out in what ways a magnetic field removes this degeneracy with respect to modes with different m but the same ℓ.

4.1 Global Fields

We will first assume that the magnetic field is a global field, so that it does not vary rapidly at smaller scales. Also, we will assume that ℓ is not too large, say $\ell \leq 10$. This seems reasonable since we have never identified any white dwarf pulsations as having $\ell \geq 3$. The purpose of these assumptions is to ensure that the radial variation of the displacement vector is more rapid than the other gradients of $\vec{\xi}$ or any gradients of $\vec{B}$. Since we have already shown that the horizontal displacements are much larger than those in the vertical direction, we in effect are only going to retain terms which are proportional to $(\frac{d}{dr}\xi_h)^2$ or $\xi_h \frac{d^2}{dr^2}\xi_h$.

Using a vector identity (Jackson 1975), we have

$$\begin{aligned}
\vec{B'} &= \nabla \times (\vec{\xi}_{klm} \times \vec{B}) \\
&= \vec{\xi}_{klm}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{\xi}_{klm}) + (\vec{B} \cdot \nabla)\vec{\xi}_{klm} - (\vec{\xi}_{klm} \cdot \nabla)\vec{B} \\
&\approx (\vec{B} \cdot \nabla)\vec{\xi}_{klm} \\
&\approx B_r \frac{d}{dr}\vec{\xi}_{klm} \\
&\approx B_r \frac{d}{dr}\xi_h(r)\, r\nabla Y_\ell^m(\theta,\phi),
\end{aligned} \tag{4.2}$$

where $B_r = B_r(r,\theta,\phi)$ is the radial component of the magnetic field. Using this form for $\vec{B'}$, we find that the only term which is proportional to $(\frac{d}{dr}\xi_h)^2$ comes from the second term in brackets in equation 3.10. Using equation 4.1, and the recasting of the second term given in equation 3.12, we see that the expression for the frequency perturbation simplifies to

$$\Delta\sigma_{k\ell m} = \frac{1}{8\pi\sigma_{kl}} \cdot \frac{\int \rho dV |\vec{B'}|^2}{\int \rho dV |\vec{\xi}_{klm}|^2}. \tag{4.3}$$

Inserting equation 4.2 and employing the magic of spherical harmonics, we can factor out the angular dependence of $\vec{B}$ in equation 4.3:

$$\Delta\sigma_{k\ell m} = \frac{1}{8\pi\sigma_{kl}} \cdot \frac{\int_0^{R_c} r^2 dr (\frac{d}{dr}\xi_h)^2 I_{\ell m}(r)}{\int_0^{R_c} \rho r^2 dr ({\xi_r}^2 + \ell(\ell+1){\xi_h}^2)}, \tag{4.4}$$

where R_c is the cutoff radius at which the gas pressure equals the magnetic pressure, and where I_{lm} is defined by

$$I_{\ell m}(r) \equiv \int d\Omega {B_r}^2(r,\theta,\phi)\left[\left|\frac{\partial}{\partial\theta}Y_\ell^m(\theta,\phi)\right|^2 + \frac{1}{\sin^2\theta}\left|\frac{\partial}{\partial\phi}Y_\ell^m(\theta,\phi)\right|^2\right]$$

$$= \int d\Omega {B_r}^2(r,\theta,\phi)\left[\ell(\ell+1)\left|Y_\ell^m(\theta,\phi)\right|^2 + \frac{1}{2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\left|Y_\ell^m(\theta,\phi)\right|^2\right)\right]$$

$$= \int d\Omega \left|Y_\ell^m(\theta,\phi)\right|^2\left[\ell(\ell+1){B_r}^2(r,\theta,\phi) + \frac{1}{2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}{B_r}^2(r,\theta,\phi)\right)\right] \tag{4.5}$$

From this, we can show that if $B_r = B_r(r)$, then there would be no m splitting since the angular integrations over the spherical harmonics would just yield 1. Such a form for $\vec{B}$ is not possible, however, since it would imply a monopole term in the expansion of the magnetic field. Consequently, in general $B_r = B_r(r,\theta,\phi)$, so we expect there to be splitting from any global magnetic field. It is also clear that all the splitting information resides in the function $I_{\ell m}$. We should note that it is not possible to obtain any information about the ϕ dependence of B_r from this formula, since $|Y_\ell^m(\theta,\phi)|$ is independent of ϕ.

It is interesting to examine $I_{\ell m}$ for particular values of ℓ and m. For $\ell = 1$, we have

$$I_{11} = \frac{3}{8\pi}\int d\Omega B_r^2(1+\cos^2\theta) \tag{4.6}$$

$$I_{10} = \frac{3}{4\pi}\int d\Omega B_r^2(1-\cos^2\theta), \tag{4.7}$$

or writing this more suggestively,

$$2I_{11} + I_{10} = \frac{3}{4\pi}\int d\Omega B_r^2 = \frac{3}{\sqrt{4\pi}}\int d\Omega B_r^2 Y_0^0 \tag{4.8}$$

$$I_{11} - I_{10} = \frac{3}{8\pi} \int d\Omega B_r^2 (3\cos^2\theta - 1) = \frac{3}{2\sqrt{5\pi}} \int d\Omega B_r^2 Y_2^0. \qquad (4.9)$$

Note that $I_{11} - I_{10}$ corresponds to the frequency splitting between the $m = 0$ and 1 modes, which is an observable quantity. Another way of thinking of equation 4.9 is that the splitting of the $\ell = 1$ modes is due to the 'quadrupole moment' of the radial contribution to the magnetic energy density. Again, since B_r must have have some angular dependence, it can be expanded in a series of spherical harmonics, which will result in non-zero splittings.

If we make the assumption that $B_r(r, \theta, \phi) = f(r) Y_{\ell'}^{m'}(\theta,\phi)$, then we obtain

$$I_{11} = \frac{3}{8\pi} f^2(r)(1 + \Delta_0) \qquad (4.10)$$

$$I_{10} = \frac{3}{4\pi} f^2(r)(1 - \Delta_0), \qquad (4.11)$$

where

$$\Delta_0 = \frac{2(\ell'(\ell'+1) - m'^2) - 1}{(2\ell' - 1)(2\ell' + 3)}. \qquad (4.12)$$

Similar formulae to equations 4.8–4.12 could be obtained for I_{22}, I_{21}, and I_{20}, of course.

If we examine equation 4.5, we see that the splitting only depends on the radial component of the magnetic field, B_r. For both a dipole field and a constant field in the $\hat{z}$ direction, $B_r \propto \cos\theta$. Hence the m dependence of the splitting for the two cases (with the above approximations) is identical. We will proceed by considering only the constant field, since it embodies all of the results with which we are concerned. It should be noted that by considering fields which are curl-free, the approximations of ignoring the first and third terms in equation 3.10 become exact. Also, there are no structural perturbations since the Lorentz force is zero, so $\delta\mathcal{L}$ is also zero, as was noted in section 3.2.

For this case, the formula for a constant field simplifies to

$$\Delta\sigma_{k\ell m} = \frac{B^2 A_{\ell m}}{8\pi\sigma_{k\ell}} \cdot \frac{\int_0^{R_c} r^2 dr (\frac{d}{dr}\xi_h)^2}{\int_0^{R_c} \rho r^2 dr (\xi_r^2 + \ell(\ell+1)\xi_h^2)},$$

where

$$A_{\ell m} \equiv \ell(\ell+1) - m^2 - \Delta_1,$$

$$\Delta_1 = \frac{2\ell^2(\ell+1)^2 - m^2(2\ell(\ell+1)+3)}{(2\ell-1)(2\ell+3)}.$$

Inserting two values for ℓ, we find

$$A_{\ell m} = \begin{cases} \frac{2}{5} + \frac{2}{5}m^2 & \ell = 1 \\ \frac{18}{7} - \frac{2}{7}m^2 & \ell = 2 \end{cases} \tag{4.13}$$

We therefore obtain the interesting result that (in this approximation) the splitting increases with m for $\ell = 1$ but decreases with m for $\ell \geq 2$. This agrees with the behavior seen by Jones et al. (1989) in their numerical results.

Figure 4.1 shows the qualitative features which a magnetic field could have on a multiplet which was already rotationally split. It is well known that rotational splitting varies linearly with m within a multiplet (Hansen et al. 1977),

$$\sigma_{klm} = \sigma_{kl} + m\,\delta\sigma_{\mathrm{rot}}, \tag{4.14}$$

whereas, from equation 4.13, we see that the magnetic splitting is proportional to m^2,

$$\sigma_{klm} = \sigma_{kl} + m^2\,\delta\sigma_{\mathrm{mag}}. \tag{4.15}$$

In the limit that both effects are perturbative, the resulting frequencies are simply due to the sum of the two effects, i.e.,

$$\sigma_{klm} = \sigma_{kl} + m\,\delta\sigma_{\mathrm{rot}} + m^2\,\delta\sigma_{\mathrm{mag}}. \tag{4.16}$$

From equation 4.13, we see that $\delta\sigma_{\mathrm{mag}}$ is positive for $\ell = 1$ and negative for $\ell = 2$ or greater. This is the source of the difference in appearance for the $\ell = 1$ and 2 splittings shown in figure 4.1.

4.2 Rapidly Varying, Disordered Fields

By a rapidly varying field we mean a field which has a large gradient, i.e.,

$$\left|\frac{\nabla B}{B}\right| \gg \left|\frac{\nabla f}{f}\right|, \tag{4.17}$$

where f is any other variable under consideration. There is reason to believe that such a field could be produced by the fluid motions in a convection zone. Given an initially smooth, global field, the velocities in the convection zone, due to flux freezing, could twist and deform the field until it had structure on scales at least as small as the individual convection cells themselves.

We will also assume that, due to processes such as these, the field is disordered in the sense that different components of $\vec{B}$ are uncorrelated. The approximations we will use to calculate the disordered nature of $\vec{B}$ are quite similar to those used to discuss velocity correlation functions in fully developed turbulence (Landau and Lifshitz 1959). For our purposes, they are

$$\langle B_i B_j \rangle \;=\; \frac{\delta_{ij}}{3}\langle B^2 \rangle \tag{4.18}$$

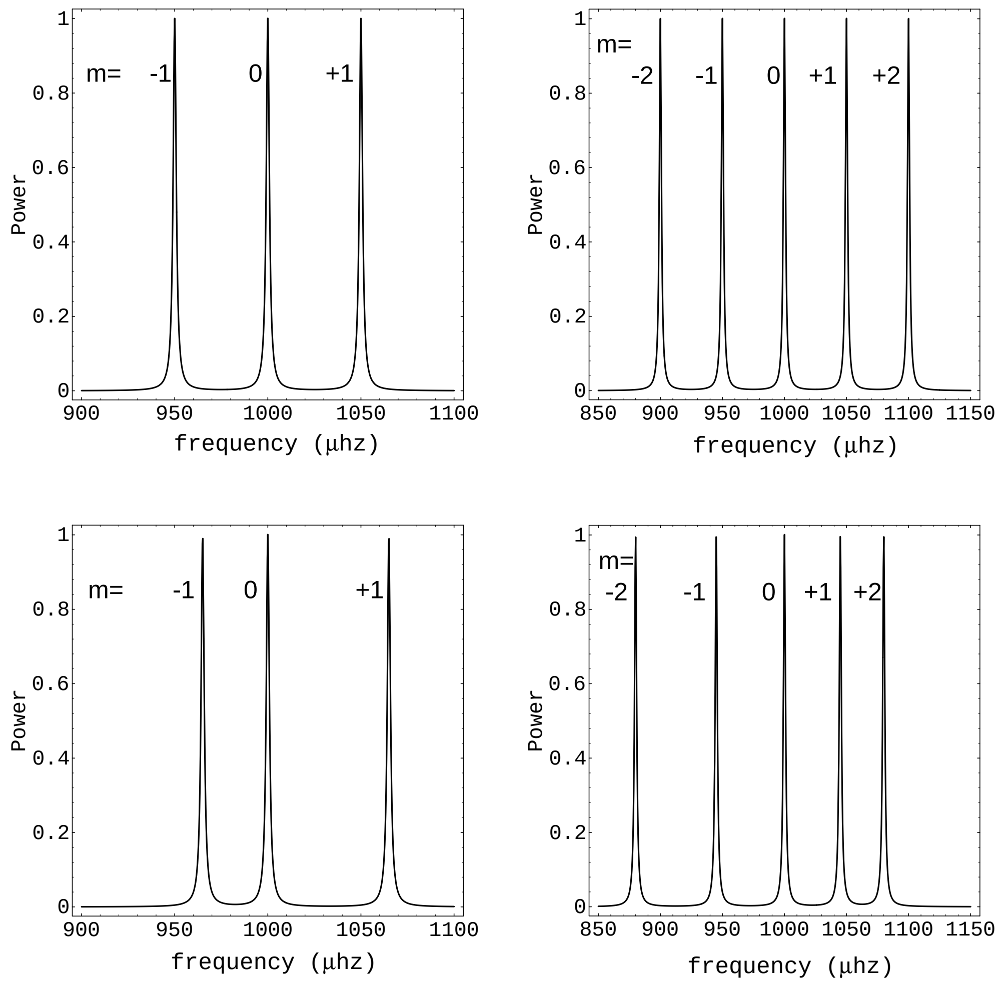

Figure 4.1: The qualitative effect of small frequency perturbations due to a magnetic field superimposed on rotationally split multiplets. The plots on the left are for $\ell = 1$, and those on the right are for $\ell = 2$. The upper graphs are without the magnetic field, and the lower ones are with it. The magnetic plots have been shifted so that the $m = 0$ mode has a frequency of 1000 μHz.

$$\begin{aligned}\langle \partial_i B_j \partial_k B_l \rangle &= \frac{\langle B^2 \rangle}{30{d_m}^2}[4\delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}] \qquad (4.19)\\ \langle B_j \partial_k B_l \rangle &= 0 \qquad (4.20)\end{aligned}$$

where d_m is a scale length for the variations of the magnetic field, $\langle\rangle$ denotes averaging over small spatial scales, and ∂_i means $\frac{\partial}{\partial x_i}$. The reason for the complicated tensor structure of equation 4.19 is to ensure that the condition $\nabla \cdot \vec{B} = 0$ is maintained.

We now wish to compute $\vec{B'}$ neglecting all derivatives except those which act on the components of $\vec{B}$. From the second line in equation 4.2 we therefore obtain

$$\vec{B'} \approx (-\xi \cdot \nabla)\vec{B} \approx -\xi_j \vec{e}_i \partial_j B_i \qquad (4.21)$$

where the $\vec{e}_i$ are the unit vectors, and summation over repeated indices is implied. Since we are not assuming any particular form for the magnetic field, we must use the general expression in evaluating $\langle \vec{\xi}^* \cdot \mathbf{B}(\vec{\xi'}) \rangle$, as is given in equation 3.10. Taking the terms on the RHS of equation 3.10 one at a time, we note that for the first term, $\langle \vec{B} \times (\nabla \times \vec{B}) \rangle = 0$, since it is only linear in derivatives. This leaves the second and third terms. The second term is

$$\begin{aligned}\langle |\vec{B'}|^2 \rangle &\approx \langle {\xi_j}^* (\partial_j B_i) \xi_k (\partial_k B_i) \rangle\\ &= {\xi_j}^* \xi_k \langle (\partial_j B_i)(\partial_k B_i) \rangle\\ &= |\vec{\xi}|^2 \frac{\langle B^2 \rangle}{3 d_m^2}, \qquad (4.22)\end{aligned}$$

and the third term evaluates to

$$\begin{aligned}\langle \vec{\xi} \cdot [(\nabla \times \vec{B}) \times \vec{B'}] \rangle &= \langle (\nabla \times \vec{B}) \cdot (\vec{B'} \times \vec{\xi}) \rangle\\ &\approx -\epsilon_{ijk}\epsilon_{ilm}\xi_m \xi_p \langle (\partial_j B_k)(\partial_p B_l) \rangle\end{aligned}$$

$$= \quad |\vec{\xi}|^2 \frac{\langle B^2 \rangle}{3d_m^2}, \tag{4.23}$$

where ϵ_{ijk} is the completely anti-symmetric Levi-Civita tensor. Adding these contributions and again using $\xi_h \gg \xi_r$, we obtain an equation identical in form to equation 4.4:

$$\Delta\sigma_{k\ell m} = \frac{1}{8\pi\sigma_{kl}} \cdot \frac{\int_0^{R_c} r^2 dr \xi_h^2 I_{\ell m}(r)}{\int_0^{R_c} \rho r^2 dr ({\xi_r}^2 + \ell(\ell+1){\xi_h}^2)}, \tag{4.24}$$

where now we have

$$I_{\ell m}(r) \equiv \int d\Omega \frac{2}{3d_m^2} \langle B^2 \rangle \left[\left| \frac{\partial}{\partial\theta} Y_\ell^m(\theta,\phi) \right|^2 + \frac{1}{\sin^2\theta} \left| \frac{\partial}{\partial\phi} Y_\ell^m(\theta,\phi) \right|^2 \right]$$

$$= \frac{2}{3d_m^2} \int d\Omega \, |Y_\ell^m(\theta,\phi)|^2 \left[\ell(\ell+1)\langle B^2 \rangle + \frac{1}{2\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \langle B^2 \rangle \right) \right]. \tag{4.25}$$

Thus, we find that if $\langle B^2 \rangle$ has no angular dependence, then there will be no splitting of the frequencies as a function of m. If, on the other hand, $\langle B^2 \rangle$ has some global angular dependence, this could lead to a partial lifting of degeneracy.

As was suggested earlier, such a field might be generated by a twisting and bending of magnetic field lines which were already present in the convection zone. If the final fields are assumed to be proportional to the original fields, then $\langle B^2 \rangle$ will have the same angular dependence as did the original B^2. For example, if the field were initially constant and in the $\hat{z}$ direction, B^2 would be independent of angle, so no splitting would be produced by the final field. If on the other hand, the initial field had a dipole dependence, i.e., $\vec{B} \propto 2\hat{r}\cos\theta + \hat{\theta}\sin\theta$, then we would have

$$\begin{aligned} \langle B^2 \rangle &\propto 4\cos^2\theta + \sin^2\theta \\ &\propto 1 + 3\cos^2\theta \end{aligned} \tag{4.26}$$

which *would* produce a splitting. More precisely, the constant 1 on the RHS of equation 4.26 would produce no splitting, but the second term $\cos^2\theta$ would produce splitting with an ℓ and m dependence exactly the same as the constant or dipole fields of the previous section.

The results of the formalism presented in this chapter are numerically implemented in the next section. We also attempt an interpretation of the multiplet splittings in the DBV white dwarf GD 358.

Chapter 5

Numerical Results

5.1 Splitting by a Uniform Field

We can now apply this formalism to an actual model of a star. The model which is used here is that for GD 358, a DBV white dwarf. The reason for this choice is that this star has been observed extensively with the Whole Earth Telescope (WET) and evidence has been found which suggests that this star may possess a magnetic field (Winget et al. 1994). In fact, these observations provided much of the motivation for this theoretical investigation.

As mentioned in chapter 2, we used an RKF pulsation code, the main body of which is described in Kawaler et al. (1985). Modifications were made which allowed us to calculate the frequency perturbations of modes due to 'weak' magnetic fields using the formalism presented in the previous chapter. The white dwarf model which was used is the one presented in Bradley and Winnget (1994).

Following the lead of Jones et al. (1989), we computed the splitting between the $m = 1$ and $m = 0$ modes for $\ell = 1$ and 2. Since the magnetic field is a completely unknown quantity, we picked a simple form for it: *a constant field in the $\hat{z}$ direction.* By specifying the z-axis, we are assuming that the pulsation and magnetic axes coincide. In this case (as well as the dipole case), an exact form can be found for the angular integrals in equation 4.3

without making any approximations about the relative sizes of the horizontal or radial components of $\vec{\xi}$ or its derivatives. These exact expressions were used in the numerical calculations, although the results deviated very little from the asymptotic forms found in equation 4.4. The results are shown in figures 5.1 and 5.2. The field was taken to have a magnitude of 10^5 Gauss.

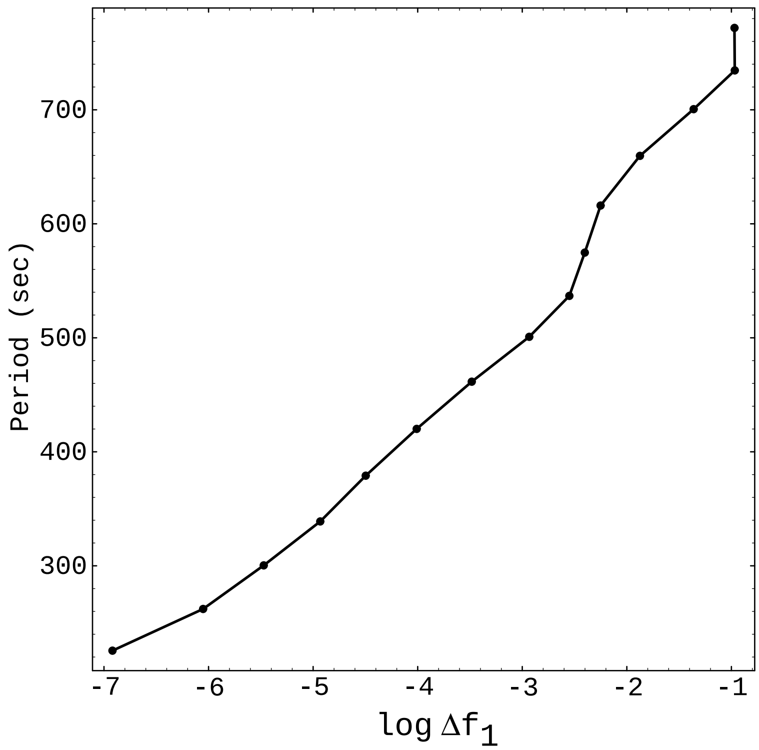

Figure 5.1: Period versus the magnitude of the frequency splitting, denoted by Δf_1, between the $m = 1$ and 0 modes, for the case $\ell = 1$.

As a general rule, modes with longer periods show larger amounts of splitting than modes with shorter periods. Since for g-modes, longer periods imply higher radial overtones (higher k), these modes have larger radial

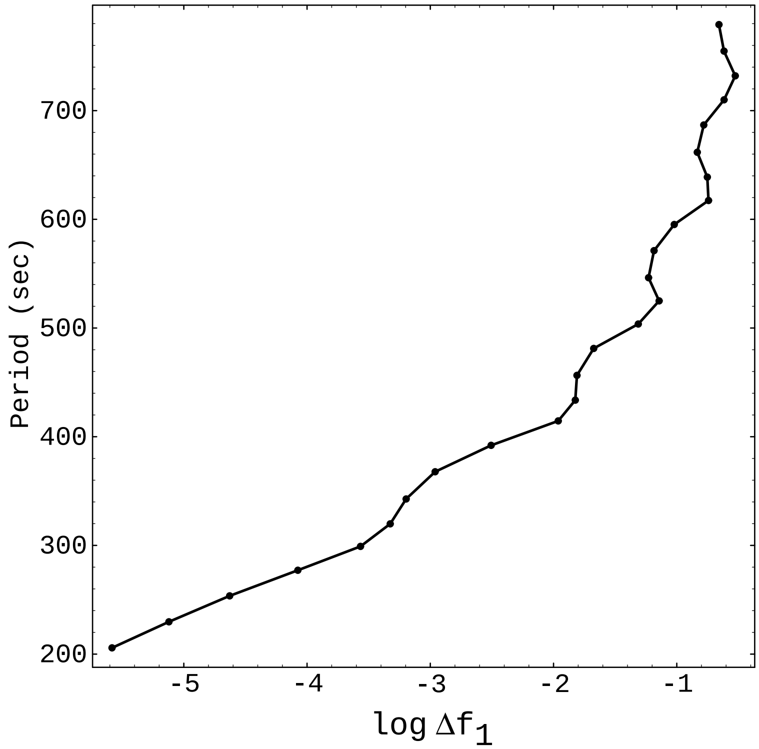

Figure 5.2: The same as figure 5.1, except for $\ell = 2$. The periodic structure seen here is a manifestation of mode trapping, i.e., the modes which show an abnormally high splitting are modes which are more localized in the surface layers ('trapped').

derivatives and hence higher splittings.

5.2 Application to the DBV GD 358

The next step is to use these calculations to interpret the asymmetric splitting in the data for GD 358 from the May 1990 WET run (Winget et al. 1994). Essentially, for each mode, we can find the magnitude of the constant magnetic field which would produce the asymmetric splitting in that individual mode. Again, we are assuming that the pulsation, magnetic, and rotation axes coincide. Since each mode may sample a different region of the star, and since that magnetic field is not likely to actually be constant in space, we expect to obtain different values of the field for each mode. Since only $\ell = 1$ modes were unambiguously identified in the data, the modes shown differ only in radial overtone number. The results of this procedure are displayed in figure 5.3.

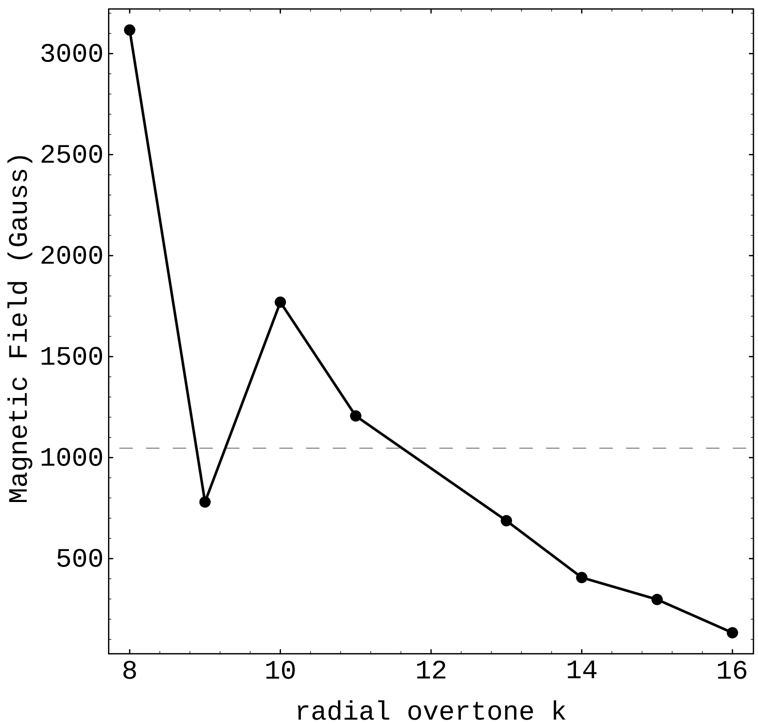

Figure 5.3: The average field 'felt' by modes of different k with $\ell = 1$, as derived from the observed splitting asymmetries in GD 358. The dotted line shows where the average ($\sim$ 1050 G) of these values lies.

Ideally, figure 5.3 would be a horizontal line, i.e., each mode would be sampling the same average magnetic field. The fact that it is not means that something more complicated must be going on. Since we know that different modes do not have the same radial dependence, we should investigate whether a field which varies as a function of radius is able to reproduce the splittings we observe. As a first step we examine the sensitivity of each of the different splittings to the different regions in the star, again for a magnetic field of the form $\vec{B} = B(r)\hat{z}$. The results of this are shown in figure 5.4.

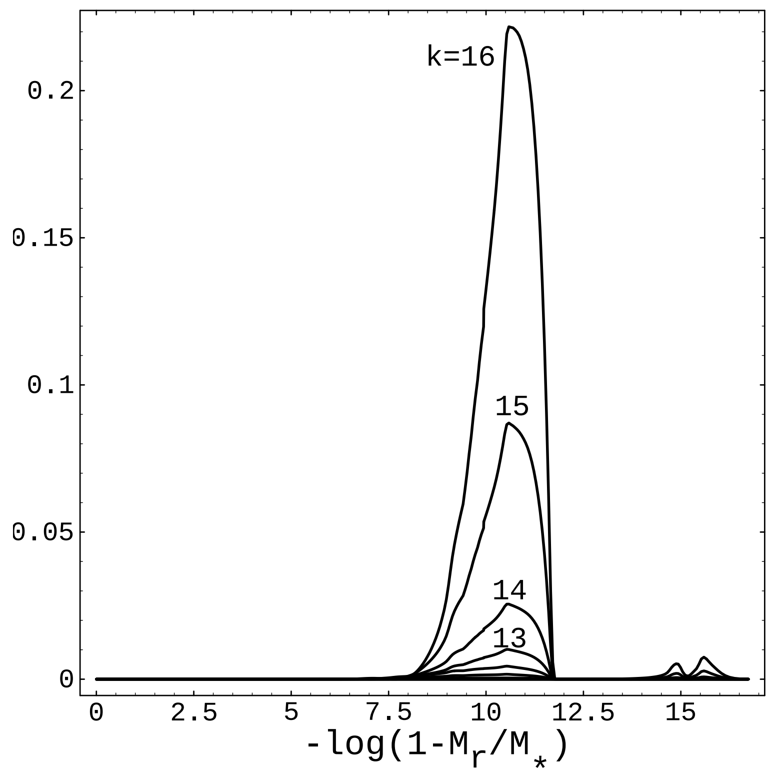

Figure 5.4: The relative sensitivity of each of the different modes to a given strength magnetic field which is uniform in the $\hat{z}$ direction.

The fact that the $k = 16$ mode is much higher than the others means that it is much more sensitive to being split than the other modes. Put another way, it requires much less of a field to be split by the amount which was observed in the data (it requires a field of less than 150 Gauss). In contrast, the $k = 13$ mode, which is much less sensitive, requires a field greater than 700 Gauss. Since both modes appear to sample the same region of the star, it is difficult to reconcile how such a disparity can occur. The situation is even more dramatic for the $k = 10$ and 11 modes. They require fields up to 1800 Gauss, which should be detected by the higher k modes.

Given this confusing state of affairs, one might suppose that the original assumption of a global field is incorrect. We therefore wish to consider the same question within the context of a small-scale, disordered field. If we assume that $\langle B^2 \rangle$ has the angular dependence given in equation 4.26, then we find that the different modes sample the star as shown in figure 5.5.

For this case, the modes have shifted the region which they sample farther out in the star, closer to the surface. Unfortunately, the $k = 16$ mode still dominates over almost all of the region in which the other modes are sensitive, which leads to the same paradoxical state of affairs as in the constant field case. It appears that the star is trying to tell us something.

5.3 Validity of Approximations

There are many ways in which the theory presented here is incomplete. First of all, the dynamical effects of convection are not included. This would affect the eigenfunctions in the outer mass layer of $10^{-12} M_\star$. Second, adiabatic eigenfunctions were used instead of nonadiabatic ones. Again, this would affect the eigenfunctions in the outer $10^{-12} M_\star$. Third, the effect of the magnetic

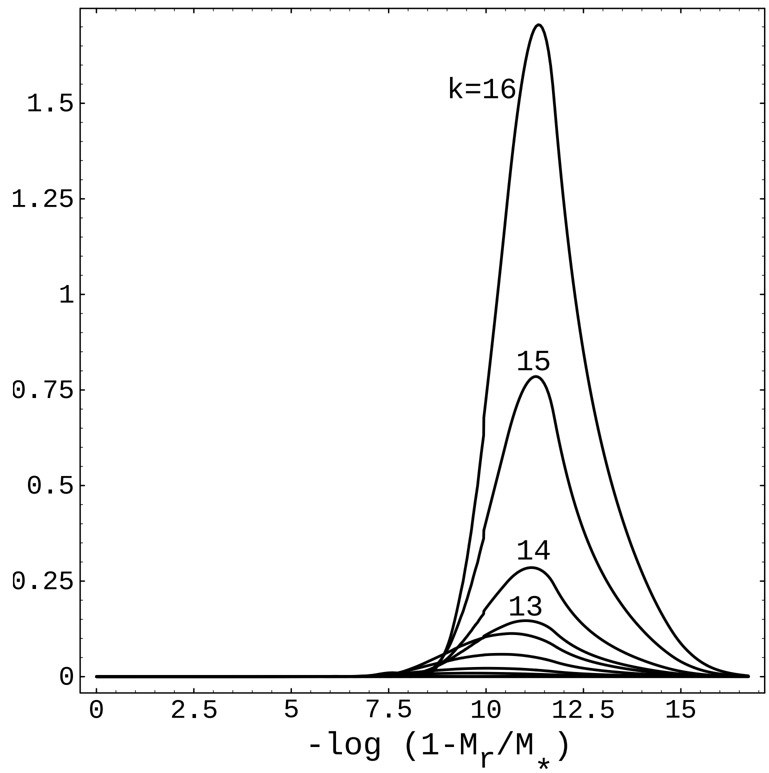

Figure 5.5: The relative sensitivity of each of the different modes to a given strength disordered magnetic field.

field on the eigenfunctions was not included, as in the case of convection; this assumption is not valid in the surface region of the star, again in approximately the outer $10^{-12}M_{\star}$.

In previous pulsational and structural studies of these stars, a region in a star's envelope as small as $10^{-12}M_{\star}$ would have virtually no effect at all on any measurable properties of the star, e.g., its luminosity or eigenfrequencies of pulsation. This would seem to be a reasonable starting point for investigations involving magnetic fields (or convection, for that matter). However, there is no reason that this must be the case; it may be that understanding the eigenfunctions in this region is the key to interpreting the data from GD 358.

5.4 A Closer Look at Convection

As mentioned in the previous section, the velocities in the convection zone have an effect on the eigenfunctions and eigenfrequencies which has thus far not been taken into account. In GD 358, these velocities are on the order of 10^5–10^6 cm/sec, which gives a turnover time on the order of a second. If we imagine that the convection region is turbulently mixed from top to bottom (this is quite plausible since the viscosity of the fluid is extremely small), then this turbulence generates an effective viscosity throughout this region.

If we take the magnitude of the viscosity to be the mixing length times the convective velocity, we obtain an effective viscosity of $\sim 10^{10}$–10^{11} cm^2/sec, for completely efficient mixing. If we add a term $\nu\nabla^2\vec{v} = i\nu\sigma\nabla^2\vec{\xi}$ to the RHS of the momentum equation, we obtain the following equation for the perturbations (assuming zero magnetic field):

$$\sigma^2\vec{\xi} = \mathcal{L}(\vec{\xi}) - i\nu\sigma\nabla^2\vec{\xi}. \tag{5.1}$$

Applying perturbation theory to this term, we find that the first order frequency corrections σ_1 are

$$\sigma_1 = -i\nu\frac{\int \rho dV \vec{\xi}^* \cdot (\nabla^2\vec{\xi})}{\int \rho dV \vec{\xi}^* \cdot \vec{\xi}}. \tag{5.2}$$

Since this correction is pure imaginary, it technically does not affect the frequency, although it will affect the width of a peak. To get a real frequency perturbation we have to go to second order in ν. If we do, we should find that

$$\sigma_2 \sim \frac{\sigma_1^2}{\sigma_0}, \tag{5.3}$$

where σ_2 is the second order frequency correction and σ_0 is the unperturbed frequency. For a 660 second $\ell = 1$ mode in GD 358, we find that σ_2 could be as large as .6 μHz. While a turbulent viscosity with no angular dependence would again produce no splitting, an angle dependent ν could produce splittings on the order of tenths of a μHz, which is exactly the magnitude splitting which we interpreted as due to a magnetic field. Depending on the details, it would appear that this effect could either mask or mimic magnetic effects.

However, we must also look at the first order correction, which is related to the coherence time of the oscillation. Since it is imaginary, it is essentially a damping exponent for this mode. In seconds, it would correspond to a damping time of 10^4–10^5 seconds for completely efficient momentum transport; this would imply that the peak in the fourier transform corresponding to the frequency of the mode would have a width of $\sim$ 10–100 μHz. Since all the peaks have widths on the order of one μHz, this viscous dissipation must not be occurring. An explanation for this is that the eigenfunctions themselves are modified by the convective region in such a way that they minimize the dissipation of energy, i.e., $\nabla^2\vec{\xi}$ is smaller in this region than it would be for the unperturbed eigenfunctions. In any case, we see that convection, within this simple framework, cannot account for the asymmetric splittings.

Convection is but one possible phenomenon which we have yet to explore in detail. There are certainly many other, perhaps more esoteric, physical effects which we have not explored or even conceived of yet.

Chapter 6

Conclusion

We have presented a general framework in which to calculate frequency shifts due to magnetic perturbations in white dwarf stars. We have found that a field with a global angular dependence is required in order to produce splittings between 'degenerate' modes. This should not be such a rare occurrence, however, since in general we expect magnetic fields to have an angular dependence; the lowest order multipole field observed in nature is a dipole field. Thus, the frequency splitting asymmetries observed in GD 358 are evidence of a magnetic field with some global structure, *if* the magnetic interpretation of the data is indeed correct.

The fact that the attempt to radially reconstruct the magnetic field was unsuccessful calls into question the interpretation of the splitting asymmetries as magnetic in origin. It is quite possible that some hitherto unexplored effect is responsible. However, it may simply be that the situation is more complicated than expected. After all, these calculations assumed that there were no velocities present in the unperturbed star; the velocities in the convection zone were not considered in the equations of oscillation, and the star was assumed not to be rotating, differentially or otherwise. Even so, it is remarkable that each triplet showed frequency asymmetries with the correct sign to be caused by magnetic splitting.

It is possible that our present models of the equilibrium structure of GD 358 are not completely correct. If GD 358 reached its present evolutionary state as the result of binary evolution, there may still be traces of its history which have yet to be erased, that is, its thermal profile may not yet be that of a white dwarf which has been cooling from the center outward; there may have been an inward flux of heat from accretion on the surface. Alternatively, it may be that the perturbations of the magnetic field associated with each mode interact with the other modes, so that a single mode feels the equilibrium field plus a field which is due to the fluid motions of the other modes. This could result in complex, time-dependent behavior, which *is* observed in the frequency spectra of many variable white dwarfs. In addition, the non-zero velocities in the convection zone could play an unknown role in the splitting, as was discussed in section 5.4.

Although the asymmetric splittings in GD 358 remain a mystery for now, we are confident that white dwarfs provide the best laboratory for the study of these phenomena. Only here do we find such simplifications as sphericity to a very high degree due to the large surface gravities, motions mainly in horizontal rather than radial directions (also due to high surface gravity), very thin convection zones, and little or no energy generation as a result of (difficult to calibrate) nuclear processes. Adding to this list the fact that some of these objects pulsate in nonradial modes, one could not in good conscience ask for a better astrophysical laboratory.

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