Mon. Not. R. Astron. Soc. **330,** L6–L10 (2002)

A-star envelopes: a test of local and non-local models of convection

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Accepted 2001 December 7. Received 2001 December 4; in original form 2001 November 21

ABSTRACT

We present results of a fully non-local, compressible model of convection for A-star envelopes. This model quite naturally reproduces a variety of results from observations and numerical simulations which local models based on a mixing length do not. Our principal results, which are for models with T_{eff} between 7200 and 8500 K, are the following. First, the photospheric velocities and filling factors are in qualitative agreement with those derived from observations of line profiles of A-type stars. Secondly, the He II and H I convection zones are separated in terms of convective flux and thermal interaction, but joined in terms of the convective velocity field, in agreement with numerical simulations. In addition, we attempt to quantify the amount of overshooting in our models at the base of the He II convection zone.

Key words: convection – stars: atmospheres – stars: interiors.

1 INTRODUCTION

Over the last five decades the most frequently used approach to describe stellar convection has been the mixing length theory (MLT) (Biermann 1948; Böhm-Vitense 1958). However, the great simplicity achieved by describing convection in terms of local variables is only attained at the cost of trade-offs, the most important of which is the specification of a mixing length that can neither be derived from rigorous theory nor from observations. More recently, turbulence models, e.g. by Canuto, Goldman & Mazzitelli (1996, hereafter the CGM model) have been used to improve the MLT expressions. These convection models still provide a local expression for the temperature gradient and contain the specification of a scalelength l. The latter also holds for non-local versions of the MLT which were proposed to account for convective overshooting. However, the intrinsic non-locality of this problem has prohibited a satisfactory solution within the context of models that use any form of local scalelength (see Renzini 1987 and Canuto 1993).

This difficulty is naturally avoided by numerical simulations which have come into use during the last decade as a tool to study stellar surface convection. Simulations in 3D have mostly been devoted to solar convection (Nordlund & Dravins 1990; Atroshchenko & Gadun 1994; Kim & Chan 1998; Stein & Nordlund 1998), while 2D simulations have been used for more extended computations over the Hertzsprung–Russell diagram (cf. Freytag 1995 and Freytag, Ludwig & Steffen 1996). Such calculations can include the entire convective part of a stellar envelope only for the case of A-stars (and some types of white dwarfs). Even then, the computational efforts become considerable, especially when realistic microphysics is used and thermally relaxed solutions are required. To use simulations for complete stellar models is thus beyond the range of present computer capabilities (cf. Kupka 2001).

Another alternative was pioneered by Xiong (1978) who used the Reynolds stress approach. This approach had previously been applied in atmospheric as well as in engineering sciences. But even in its most recent version (Xiong, Cheng & Deng 1997) his formalism still uses a mixing length to calculate the dissipation rate ϵ of turbulent kinetic energy. Canuto (1992, 1993) and Canuto & Dubovikov (1998, hereafter CD98) abandoned the use of a mixing length in their Reynolds stress models. These models provide both the mean quantities of stellar structure (temperature T, pressure P, luminosity L, and mass M or radius r) as well as the second-order moments (SOMs) of temperature and velocity fields created by stellar convection (turbulent kinetic energy ρK, temperature fluctuations $\overline{\theta^2}$, convective flux $F_{\mathrm{C}} = c_{\mathrm{p}}\rho\overline{w\theta}$, vertical turbulent kinetic energy $\frac{1}{2}\rho\overline{w^2}$, and the dissipation rate ϵ) as the solution of coupled, non-linear differential equations. Their models are thus fully non-local on the level of second-order moments. Numerical solutions of these models for the case of idealized microphysics have been presented by Kupka (1999a) and Kupka (1999b, 2001). The same equations, using realistic microphysics, were later solved for the He II convection zone of A-stars (Kupka & Montgomery 2001; these results were first discussed in Canuto 2000).

In this paper, we present solutions for complete A-star envelopes. Numerically, this problem is easier than that of convection in the Sun since A-stars are hotter and therefore have

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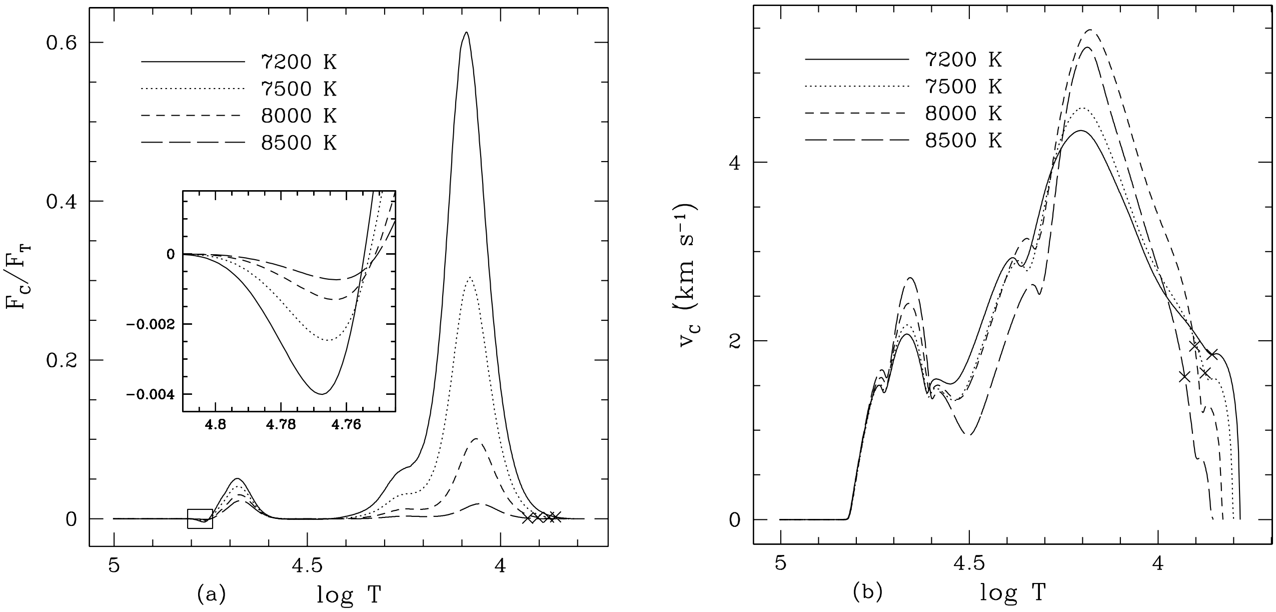

Figure 1. (a) The fraction of the flux carried by convection for four models with the indicated effective temperatures, where we have taken $\log T$ as our radial variable; $\log g = 4.4$ and $Z = 0.02$ for all the models. The cross on each curve near $\log T \sim 3.9$ shows the location where $\tau = 2/3$ for each model, and the inset is an enlargement of the indicated overshooting region. Convection becomes more dominant with decreasing $T_{\rm eff}$, but the two convection zones remain well-separated in terms of the convective flux. This holds also for $\overline{\theta^2}$ (cf. Kupka, in preparation). (b) The same as (a) but for the rms convective velocities. In contrast, we see that the convection zones *are* connected in terms of the velocity field (and also in terms of K and ϵ, see Kupka, in preparation). Thus, the two zones can be thought of as being separate thermally but not dynamically. In addition, we see that the photospheric velocities all lie in the range 1.5–2 km s^{-1}, in agreement with the lower limit of derived micro- and macroturbulence parameters (Varenne & Monier 1999; Landstreet 1998).

thinner convection zones. In addition, A-stars reveal the shortcomings of local convection models more clearly, as their less efficient convection is much more sensitive to details in the modelling. Depending, for example, on whether an α of 0.5 or 1.5 is chosen in MLT for a main-sequence star with $T_{\rm eff} \sim 7500$ K, an envelope may either have a mostly radiative temperature gradient or still contain a nearly adiabatic region. This holds for any of the convection models which rely on a convective scalelength. Hence, the efficiency of convection in the envelopes of A-stars has remained an open problem and makes them a logical as well as a promising starting point for our study.

In the following, we give an outline of the physics and the numerical procedure used to compute our envelope models (the discussion of the moment equation formalism is self-contained, so that readers unfamiliar with it can skip ahead without difficulties). Results are then presented for a sequence of models which differ from each other only in $T_{\rm eff}$. We include a model with lower gravity in order to illustrate the effect of a change in $\log g$. Finally, we show that the non-local convection model agrees with the known observational constraints and the results of numerical simulations, whereas local models are fundamentally unable to do this.

2 DESCRIPTION OF MODEL

The convection model used here is an extension of the CD98 model which requires the solution of five differential equations of first order in time and second order in space for K, $\overline{\theta^2}$, $J = \overline{w\theta} = F_C/(\rho c_p)$, $\overline{w^2}$, and ϵ, and of an additional equation for the time evolution of T [cf. equations (1)–(5) and (8) in Kupka 1999b]. This system is completed by an equation for the total pressure ['hydrostatic equilibrium' including turbulent pressure, equation (7) in Kupka 1999b] and for the mass ('conservation of mass'). We solve this set of differential equations on an unequally spaced mass grid, with the zoning chosen so as to resolve the gradients in the various quantities.

Compared with the model discussed in Kupka (1999b) the following changes and extensions have been included: (a) instead of using high Peclet number limits we apply the full form of the CD98 model for the SOMs. We thus take advantage of a better theoretical underpinning of the influence of radiative loss rates on two time scales in the equations for $\overline{\theta^2}$ and J, τ_θ and $\tau_{p\theta}$, which are provided by a well-tested turbulence model (see CD98 for a summary). (b) The Prandtl number is set to 10^{-9} as a typical value for the outer part of A-star envelopes (values up to 2 orders of magnitude larger than this do not alter our results). (c) With the exception of the pressure correlations $\overline{p'w}$ and $\overline{p'\theta}$ which require further study (see Kupka & Muthsam, in preparation), the complete form of the 'compressibility terms', equations (42)–(48) of Canuto (1993), is used to extend the CD98 model to the non-Boussinesq case. Hence, we now also include the effect of a non-zero gradient in the turbulent pressure $p_{\rm turb}$ on the superadiabatic gradient β. (d) We use a more advanced model for the third-order moments (TOMs) published in Canuto, Cheng & Howard (2001), although with a different form for the fourth-order moments (see Kupka, in preparation). If, instead, the original form for the fourth-order moments is used, the models with $T_{\rm eff} \geq 8000$ K, discussed in Fig. 1, show less efficient convection, with the opposite being true for the cooler models. In both cases, however, the results are qualitatively the same as the results we present here. As in Kupka & Montgomery (2001), we use a relation similar to equation (37f) in Canuto (1992) and thus avoid a downgradient approximation for the flux of ϵ [such as equation (6) of Kupka 1999b]. (e) The effect of stratification on the pressure correlation time-scales, τ_{pv} and $\tau_{p\theta}$, was accounted for following Canuto et al. (1994). Likewise, the time-scales τ_θ and $\tau_{p\theta}$ include a correction for the optically thin regime of stellar photospheres (cf. Spiegel 1957), while for consistency, the expression for the radiative flux F_r was taken from the stellar structure code we use for our initial models and boundary conditions (see Pamyatnykh 1999, it assumes the diffusion approximation for $\tau \geq 2/3$, but differs from it by a 'dilution factor' for optical depths $\tau < 2/3$).

More details on these alterations and comparisons with numerical simulations are discussed in Kupka (in preparation) and in Kupka & Muthsam (in preparation). With one exception we have used the original constants of Canuto (1993), CD98, and Canuto et al. (2001). We consider their adjustment to be of little use, because in case of failure it is usually the entire shape of the functional relation which is at variance with measurements or simulations (cf. the MLT example in Section 4). The one exception we have made is the high efficiency limit of $\tau_{p\theta}$, for which the CD98 model appears to predict values too low in comparison with simulations for idealized microphysics (see Kupka 2001), and also in comparison with a previous model (Canuto 1993). Most likely this is due to an isotropy assumption in its derivation and we thus use a $\tau_{p\theta}$ increased by a factor of 3 as suggested in Kupka (2001). This problem will be thoroughly discussed in Kupka & Muthsam (in preparation).

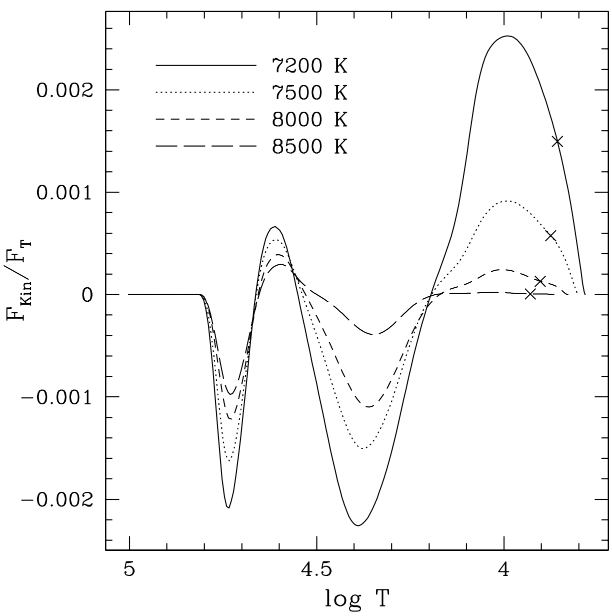

Figure 2. The kinetic energy flux as a function of $\log T$, for the four different models from Fig. 1. As expected, the cooler models have larger fluxes. Most significantly, however, these numbers show that $|F_{\rm kin}|$ is essentially negligible for the models we have examined, in contrast to the case of the Sun, where it may be as large as 20 per cent (cf. Stein & Nordlund 1998; Kim & Chan 1998).

A numerical approach to solve the resulting system of equations was briefly described in Kupka (1999a,b); a comprehensive discussion of the code will be given in Kupka (in preparation). Here we only outline the solution procedure from the viewpoint of stellar structure modelling. We start from an envelope model computed with the code described in Pamyatnykh (1999), where the equation of state and opacity data are from the OPAL project (Rogers, Swenson & Iglesias 1996; Iglesias & Rogers 1996). The metallicity, $T_{\rm eff}$, surface $\log g$, and total stellar radius R_* are taken from this model and held constant during relaxation. We place some 200 mass shells from the mid photosphere (with $\tau_{\rm ross} \sim 10^{-3}$) down to well below the He II convection zone. Having embedded the convection zones within stably stratified layers, we can use the boundary conditions of Kupka (1999b) for the SOMs (cf. Kupka, in preparation). For the mean structure quantities we keep r, T, and P fixed to their values at the upper photosphere of the input model, while a constant L is enforced at the bottom. The complete system is integrated in time (currently by a semi-implicit method) until a stationary, thermally relaxed state is found. The mass shells can be rezoned to a different relative size to resolve, e.g., steep temperature gradients that may appear and/or disappear during convergence. The radiative envelope below the convection zones may then be obtained from a simple downward integration.

Generating a complete stellar model would require fitting such an envelope on to a stellar core, which in turn requires iterating the envelope parameters ($T_{\rm eff}$, $\log g$, R_*) to achieve a match of P, T, and r at the core/envelope interface. Since we have not yet computed evolutionary models, we have not needed to do this, although this would be a straightforward extension of our work.

3 RESULTS

Fig. 1 shows the central results of this paper: both the He II and H I convection zones appear quite separate when the quantity which is examined is the convective flux (a), but completely merged in terms of the convective velocity field (b). Thus, to obtain a self-consistent solution, one must solve the equations for the entire region simultaneously.

From Fig. 1(a) (and Fig. 2), we see that the mid to the upper photospheres of these models (the crosses indicate the point where $\tau = 2/3$) are essentially radiative, as they are in the local CGM and MLT models. Thus, the temperature and density structure of both the local and non-local models are virtually identical at small optical depths, which justifies our use of the local models as an outer boundary condition for the non-local models.

In Table 1, we list these results. Since the He II and H I convection zones are well-separated in terms of F_C/F_T, we have listed their maximum fluxes separately (columns 3 and 4). For the convective velocity, $v_C = (\overline{w^2})^{0.5}$, we have listed just a single maximum since this quantity is large throughout the entire region

Table 1. Convection zone parameters obtained with the non-local model. The overshooting (OV) is measured from the minimum of F_C/F_T (shown in the inset of Fig. 1a) to the point where $|F_C/F_T| \sim 10^{-6}$.

$T_{\rm eff}$	$\log g$	$(F_C/F_T)_{\rm max}$		OV	$(v_C)_{\rm max}$	$(v_C)_{\tau=2/3}$	$(p_{\rm turb}/p_{\rm tot})_{\rm max}$	$(p_{\rm turb}/p_{\rm tot})_{\tau=2/3}$
(K)		He II	H I	(in H_p)	(km s^{-1})	(km s^{-1})		
8500	4.4	0.023	0.019	0.44	5.29	1.60	0.131	0.043
8000	4.4	0.030	0.100	0.46	5.48	1.94	0.146	0.068
7500	4.4	0.041	0.303	0.45	4.61	1.64	0.105	0.053
7200	4.4	0.051	0.612	0.46	4.36	1.85	0.100	0.069
6980	3.53	0.038	0.164	0.52	5.33	1.40	0.130	0.042

(column 6); the same holds for the relative turbulent pressure (column 8). Since all of these maxima occur below the stellar surface, we have also listed the photospheric ($\tau = 2/3$) values of v_C and p_{turb}/p_{tot} (columns 7 and 9).

Finally, in Fig. 2 we plot the kinetic energy flux as a function of $\log T$, for the four different models from Fig. 1. Besides the fact that the cooler models have larger fluxes, which is to be expected, we see from the magnitudes of these fluxes that F_{kin} is essentially negligible for the models we have examined. We are thus in a different regime from that of the Sun, where $|F_{kin}/F_T|$ may be as large as 20 per cent (cf. Stein & Nordlund 1998, Kim & Chan 1998).

In addition to these results, we have also run low- and high-metallicity models ($Z = 0.006$, 0.06, respectively). We find that for the low-Z models, $(v_C)_{max}$ decreases $\lesssim 3$ per cent while $(F_C)_{max}$ increases by $\lesssim 10$ per cent, with the opposite trends for the high-Z models. While these changes are not large, we note that they would be enhanced by the use of non-grey atmospheres. On the other hand, reducing $\log g$ (to a value still consistent with a main-sequence object) results in much weaker convection caused by a lower density and hence smaller heat capacity of the fluid, as shown by the last model in Table 1, which is taken from an (MLT based) evolutionary sequence of a 2.10-$M_\odot$ star.

4 DISCUSSION

The fact that F_{kin} is positive in the photosphere for each of these models (Fig. 2) means that the skewness of spectral lines produced in this region is also positive, and that the corresponding filling factors for rising versus falling fluid elements is less than 1/2 (cf. CD98). This is in agreement with the observations of line profiles in A-stars (Landstreet 1998). In the future, quantitative comparisons with such observational data will provide some of the most stringent tests of this model.

As previously mentioned, the He II and H I convection zones may be thought of as being thermally disconnected but dynamically coupled, a situation which is impossible within the context of MLT (or CGM) models. A further shortcoming of MLT is shown in Fig. 3. The convective flux of two MLT models, with mixing lengths of $l = 0.36$ and $0.42H_p$, respectively, is plotted along with the flux from the non-local solution (upper panel). First, we see that it is impossible for the MLT models to match simultaneously the flux in both the He II and H I convection zones (at least with the same mixing length). Secondly, even if we try to model only the H I convection zone, fixing the mixing length so as to match the maximum flux results in a convection zone which is much too narrow. In addition, this produces photospheric velocities which are ~ 3 orders of magnitude smaller than those of the non-local model and the observations (lower panel, Fig. 3, see also Section 5). We note that since the upper photosphere is optically thin and therefore locally stable against convection, local convection models will always predict convective fluxes which are extremely small (or zero), even for values of α which are 'unreasonably' large.

As a further test of our results we have compared them with 2D simulations by Freytag (1995), Freytag et al. (1996), and additional models provided by Freytag (private communication). We find agreement with the following results from our calculations. (a) Models over the entire range of A-type main-sequence stars with T_{eff} up to 8500 K have their H I and He II convection zones dynamically connected. The vertical mean velocities in the overshoot regions around $\log T \sim 4.4$ are of order 1.5 to 3 km s^{-1}.

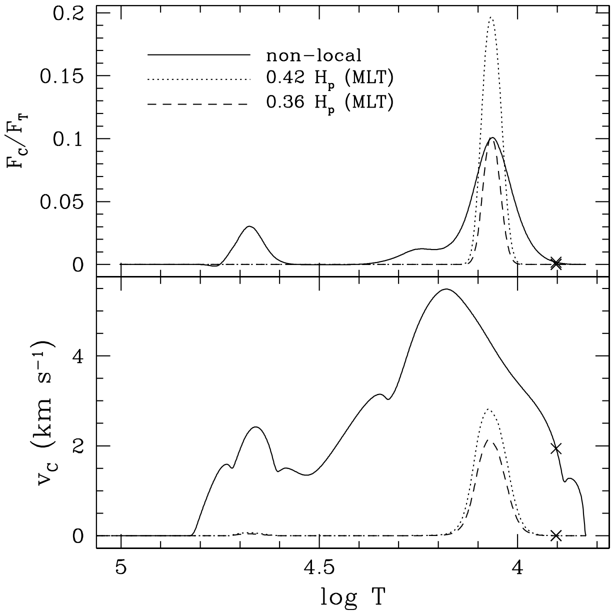

Figure 3. A comparison of the convective fluxes (upper panel) and velocities (lower panel) for the non-local model and for two MLT models ($l = 0.36$ and $0.42H_p$), for $T_{eff} = 8000$ K. We see that the MLT models are unable to match simultaneously the flux in the H I and He II convection zones, or even to match both the maximum flux and the width of just the H I convection zone. In addition, the photospheric velocities of the MLT models are ~ 3 orders of magnitude smaller than those of the non-local model.

(b) There is considerable overshooting (OV) below the He II convection zone. However, the 2D simulations yield a size of the OV region which is three times larger in terms of radius and also much larger in terms of H_p, for the entire range of models in Fig. 1 and Table 1. Such differences are anticipated from a comparison of numerical simulations in 2D and 3D (Muthsam et al. 1995, see also fig. 1 in Kupka 2001). More detailed examples demonstrating that 2D simulations yield upper limits for the (3D) OV extent will be given in Kupka & Muthsam (in preparation). (c) The maximum of F_C and the temperature gradient in the H I convection zone for the models with $T_{eff} \geq 8000$ K are in good agreement with those of the 2D simulations. However, for the models with lower T_{eff}, the 2D simulations yield higher convective fluxes and lower temperature gradients and, hence, the two convection zones merge thermally at a T_{eff} which is ~ 200 K to ~ 300 K higher than in our non-local models.

Apart from the differences between 2D and 3D convection, one important reason for discrepancies is the effect of ionization (cf. also Kupka, in preparation). Briefly summarized, the current convection model assumes an ideal gas equation of state for the purpose of computing the ensemble averages in the expression for the convective flux. Using an improved, although approximate, expression for the convective (enthalpy) flux, we estimate that this assumption introduces errors of order 15–20 per cent in the convective flux. Finally, a potentially significant source of discrepancies between our models and the 2D simulations is the use of a different equation of state and opacities (OPAL, Rogers et al. 1996 versus ATLAS6, Kurucz 1979) and the non-diffusive law we use for the photospheric radiative flux (see Section 2). This does not allow us to make a detailed quantitative comparison of model sequences. Thus, we have had to restrict ourselves to only a qualitative discussion.

5 CONCLUSIONS

Using a fully non-local, compressible convection model together with a realistic equation of state and opacities, we have calculated envelope models for stellar parameters appropriate for A-stars. In examining the results of this model, we have found many points of agreement both with observations and with numerical simulations.

First, our photospheric velocities are consistent with the lower limit of the typical micro- and macroturbulence parameters found for A-stars ($1.5-2\,\mathrm{km\,s^{-1}}$, see Varenne & Monier 1999 and Landstreet 1998). Line blanketing should further increase these values. We expect a smoother $v_{\mathrm{C}}(r)$ (without small minima as in Figs 1b and 3) from an improved treatment of fourth-order moments and inclusion of $\overline{p'w}$ (cf. Section 2). Secondly, we find that the filling factor for rising fluid elements in the photospheres of our models is less than 1/2, also in agreement with observations of line profiles in A-stars. Thirdly, we find in the temperature range 7200 to 8500 K that the He II and H I zones are well-separated in terms of the convective flux but *not* in terms of the convective velocity field. The two zones are thus in some sense thermally separated but dynamically joined. This feature is also shown by the numerical simulations. Finally, we find an OV at the base of the He II convection zone of $\sim 0.45H_p$. The numerical simulations find an even larger OV, but this may also be due to the fact that they were done in 2D. We note that in all cases we find a nearly radiative temperature gradient in the OV region, whereas the velocities in this region remain quite large, within an order of magnitude of their maxima within the convection zone ($\sim 0.5\,\mathrm{km\,s^{-1}}$).

In addition, the non-local model yields smaller temperature gradients than the local model of Canuto et al. (CGM, 1996). Such a comparison with MLT is more difficult due to the large range of α in current use. Nevertheless, we have found evidence that for main-sequence models α has to be decreased from values of ~ 1.0 at about 7100 K to ~ 0.4 for models with $T_{\mathrm{eff}} = 8000\,\mathrm{K}$ in order to obtain a comparable value of $(F_{\mathrm{C}})_{\mathrm{max}}$ in the H I convection zone. In order to match $(F_{\mathrm{C}})_{\mathrm{max}}$ in the He II convection zone, a completely different set of α's (with larger values) would be required.

As already mentioned, A-stars are excellent choices for this first calculation since they have relatively thin surface convection zones, so that the thermal time-scales involved are not so long. In addition, they are interesting stars in their own right, containing high-metallicity stars (the Am stars) as well as two groups of pulsating stars (the roAp and δ Scuti stars). In the future, it may be possible to use the pulsating stars as probes of the subsurface convection zones, much as has been done in the case of the Sun.

ACKNOWLEDGMENTS

This research was performed within project *P13936-TEC* of the Austrian Fonds zur Förderung der wissenschaftlichen Forschung (FwF), and was supported by the UK Particle Physics and Astronomy Research Council. We thank Dr B. Freytag for providing us with results from his simulations.

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